A CHARACTER of a group $G$ in a field $\mathbb{K}$ is a group homomorphism $G \to \mathbb{K}^*$. If $\chi_1, \ldots, \chi_m$ are distinct characters of $G$ in $\mathbb{K}$, prove they are linearly independent (as functions) over $\mathbb{K}$, i.e., there is no non-trivial relation $\sum_{i=1}^{m} \alpha_i \chi_i = 0$, $\alpha_i \in \mathbb{K}$.

Hint: This is related to one of the arguments used in Reed FTGT.