1. Determine all conjugacy classes in $S_n$ for $n \leq 4$.

2. Determine all subgroups in $A_4$. Show that $A_4$ has no subgroup of order 6.

3. (a) Prove that $S_n$ is generated by $(1, 2), (1, 3), \ldots (1, n)$.
    (b) Prove that $S_n$ is generated by two cycles $(1, 2)$ and $(1, 2, \ldots, n)$.

4. Show that $A_n$ ($n \geq 4$) and $S_n$ ($n \geq 3$) have trivial centers.

5. (a) Show that the centralizer of $A_n$ in $S_n$ (the subgroup in $S_n$ consisting of all elements, which commute with all elements in $A_n$) is trivial, if $n \geq 4$.
    (b) Let $g \in S_n$ be an odd transformation. Show that the map $f : A_n \rightarrow A_n$, given by $f(x) = gxg^{-1}$, is an automorphism. Prove that $f$ is not inner automorphism if $n \geq 3$.

6. Prove that every automorphism of $S_3$ is inner and $Aut(S_3)$ is isomorphic to $S_3$.

7. Describe all Sylow subgroups in $S_5$.

8. Show that every subgroup in $S_n$ of index $n$ is isomorphic to $S_{n-1}$. (Hint: For a subgroup $H \subset S_n$ of index $n$ consider the homomorphism $S_n \rightarrow S_X$, where $X = S_n/H$, induced by the action of $S_n$ on $X$ by left translations.)

9. (a) Show that for $n \neq 4$, any proper subgroup in $A_n$ has index $\geq n$. (Hint: See the hint to problem 8.)
    (b) Prove that there are no injective homomorphisms $S_n \rightarrow A_{n+1}$ for $n \geq 2$.

10. (a) Show that for any $n \geq 1$ there is an injective homomorphism $S_n \rightarrow A_{n+2}$.
     (b) Prove that any finite group is isomorphic to a subgroup of a finite simple group.