1. (20 pt.) Exercise 17 (a) in Section 2.2 of the textbook.

2. (20 pt.) Write down sentences (in the language introduced in Problem Set 4, Exercise 7) that express the axioms for vector spaces over \( \mathbb{Q} \).

3. (20 pt.) Consider a first-order language containing the 2-place predicate symbol \( R \). Write down an \( S \)-formula which expresses that \( R \) is the graph of a 1-place function. (The graph of a 1-place function \( f : A \rightarrow B \), where \( A \) and \( B \) are sets, is the set \( \{(a,b) \in A \times B : b = f(a)\}\).

4. (40 pt.) Consider the first-order language with a single 2-place predicate symbol \( E \). A structure \( \mathcal{G} = (G, E^\mathcal{G}) \) is called an undirected graph if
   \[ \mathcal{G} \models \forall x \neg Exx, \quad \mathcal{G} \models \forall x \forall y (Exy \leftrightarrow Eyx). \]
   The elements of \( G \) are called the vertices of \( \mathcal{G} \). We visualize a graph \( \mathcal{G} = (G, E^\mathcal{G}) \) by thinking of its vertices as points in the plane, with vertices \( a \) and \( b \) satisfying \( (a,b) \in E^\mathcal{G} \) connected by a line (called an edge of \( \mathcal{G} \)).
   (a) Describe
   \[
   \begin{array}{c}
   \bullet
   \\
   \bullet
   \\
   \bullet
   \\
   \bullet
   \\
   \bullet
   \\
   \bullet
   \end{array}
   \]
   as a structure \( \mathcal{G} \).
   (b) Prove or disprove: for every assignment \( s \) for \( \mathcal{G} \) as in (a) we have \( \mathcal{G} \models \varphi[s] \), where \( \varphi \) is the formula
   \[(Exy_1 \land Exy_2 \land Exy_3 \land Exy_4 \rightarrow \]
   \[ y_1 = y_2 \lor y_1 = y_3 \lor y_1 = y_4 \lor y_2 = y_3 \lor y_3 = y_4 )\]
   (c) Show that the following (undirected) graphs are not isomorphic (see p. 97 in the textbook):
   i. \( \mathcal{A} = \begin{array}{c}
   \bullet
   \\
   \bullet
   \end{array} \) and \( \mathcal{B} = \begin{array}{c}
   \bullet
   \end{array} \)
   ii. \( \mathcal{A} = \begin{array}{c}
   \bullet
   \\
   \bullet
   \end{array} \) and \( \mathcal{B} = \begin{array}{c}
   \square
   \end{array} \)
   iii. \( \mathcal{A} = \begin{array}{c}
   \bullet
   \end{array} \) and \( \mathcal{B} = \begin{array}{c}
   \bullet
   \end{array} \)
(d) Are the following graphs isomorphic?

5. (20 pt. extra credit.) Exercise 17 (b) in Section 2.2 of the textbook.