1. Do problems 3.2.2, 3.2.8, 3.2.12, 3.2.23, 3.2.38 in the textbook.

2. Do problems 3.3.9, 3.3.10, 3.3.14, 3.3.19, 3.3.33, 3.3.39 in the textbook.

3. Prove that the Binomial Theorem holds in any commutative ring $R$ with identity: if $n \geq 1$ and $a, b \in R$, then

$$(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k.$$  

Here we set $a^0 := 1$ for any $a \in R$, and

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = n(n-1) \cdots (n-k+1) \quad \text{for } 0 \leq k \leq n.$$  

Also, for a positive integer $n$ and $a \in R$, $na$ denotes the element $a + a + \cdots + a$ ($n$ many $a$’s) of $R$.

Hint: you may use that

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k} \quad \text{for all } 1 \leq k \leq n.$$  

4. Let $R$ be a commutative ring with identity. An element $a$ of $R$ is called nilpotent if $a^n = 0$ for some $n \geq 1$.

(a) Determine the nilpotent elements of $\mathbb{Z}$.

(b) Determine the nilpotent elements of $\mathbb{Z}_{12}$.

(c) Let $a, b, c \in R$ where $a$ and $b$ are nilpotent. Show that then $a + b$ and $ac$ are nilpotent. (Hint: you may use Problem 3.)