First Name: ___________________    ID#:____________________

Last Name: ___________________

Section: _____________________  =  \begin{cases} 
1a & \text{Tuesday with N. Cook} \\
1b & \text{Thursday with N. Cook} \\
1c & \text{Tuesday with G. Tran} \\
1d & \text{Thursday with G. Tran} \\
1e & \text{Tuesday with H. Xu} \\
1f & \text{Thursday with H. Xu} \\
\end{cases}

Rules.

• There are **FOUR** problems; ten points per problem.

• No calculators, computers, notes, books, crib-sheets,...

• Use the backs of the pages. There are two spare pages at the back.

• Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring,... Try to sit still.

• Turn off your cell-phone, pager,...

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(1) (a) Finish this definition:

   We say that $T: \mathbb{R}^m \to \mathbb{R}^n$ is a linear transformation if and only if ...

Now consider the linear system

$$
\begin{bmatrix}
0 & 1 & 0 & 7 \\
1 & 2 & 2 & 1 \\
0 & 0 & 0 & 4 \\
3 & 3 & 3 & 3 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
12 \\
12 \\
\end{bmatrix}
$$

(b) Find all solutions to this system by Gauss-Jordan elimination.
(c) Indicate the reduced row echelon form of the coefficient matrix.
(d) Is the coefficient matrix invertible? YES/NO?
(2) The following matrix is in reduced row echelon form:

\[
A = \begin{bmatrix}
1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(a) Fill in the missing numbers: \(A\) is a _____ × _____ matrix. It has rank _____.
(b) Indicate the pivots (by circling them).
(c) Is there a choice of \(\vec{b}\) so that \(A\vec{x} = \vec{b}\) has no solutions \(\text{YES/NO}\)?
    If yes, give an example of such a \(\vec{b}\).
(d) Is there a choice of \(\vec{b}\) so that \(A\vec{x} = \vec{b}\) has a unique solution \(\text{YES/NO}\)?
    If yes, give an example of such a \(\vec{b}\).
(e) Is there a choice of \(\vec{b}\) so that \(A\vec{x} = \vec{b}\) has infinitely many solutions \(\text{YES/NO}\)?
    If yes, give an example of such a \(\vec{b}\).
(3) Let $R$ and $T$ be linear transformations mapping $\mathbb{R}^2 \to \mathbb{R}^2$.
(a) $R$ is reflection across the line through the points $(0, 0)$ and $(1, 1)$.
What is the matrix representing $R$?
(b) Given that
$$T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \quad \text{and} \quad T(\begin{bmatrix} 3 \\ 3 \end{bmatrix}) = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$
What is the matrix representing $T$?
(c) Find the matrix representing the composition $R \circ R$
(d) Find the matrix representing the composition $R \circ R^{-1} \circ T \circ R$
(e) Do $R$ and $T$ commute? YES/NO?
The linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 4 & 6 & 5 \end{bmatrix}$$

(a) Determine $A^{-1}$

(b) For each $p \in \mathbb{R}$ find $\vec{x} \in \mathbb{R}^3$ so that $T(\vec{x}) = \begin{bmatrix} p \\ 0 \\ p^2 \end{bmatrix}$. 

extra paper
extra paper