(1) We take two cards (without replacement) from a well-shuffled standard deck of 52 cards. Let $X$ denote the number of these two cards that are aces and let $Y$ denote the number that are hearts.
   (a) Tabulate the joint PMF for $X$ and $Y$.
   (b) Compute the PMF for $Y$ both directly and as a marginal of the above (this provides a check on your computations).
   (c) What is the covariance of $X$ and $Y$?

(2) Each of $n$ people (whom we label 1, 2, \ldots, $n$) are randomly and independently assigned a number from the set \{1, 2, 3, \ldots, 365\} according to the uniform distribution. We will call this number their birthday.
   (a) Describe a sample space $\Omega$ for this scenario.
   Let $j$ and $k$ be distinct labels (between 1 and $n$) and let $A_{jk}$ denote the event that the corresponding people share a birthday. Let $X_{jk}$ denote the indicator random variable associated to $A_{jk}$.
   (b) Write $A_{12}$ as a subset of $\Omega$.
   (c) Tabulate the joint PMF for $X_{12}$ and $X_{13}$. Compute the PMF for the product $X_{12}X_{13}$.
   (d) Tabulate the joint PMF for $X_{12}$ and $X_{34}$. Compute the PMF for the product $X_{12}X_{34}$.
   (e) Are $A_{12}$ and $A_{34}$ independent? Are they independent conditioned on $A_{13}$?
   (f) Are $A_{12}$ and $A_{13}$ independent? Are they independent conditioned on $A_{23}$?
   (g) Compute the expected number of pairs of people who share a birthday (hint: write this the number as a sum of $X_{jk}$s).
   (h) Compute the second moment and variance of the number of pairs of people who share a birthday.

(3) My dryer contains three pairs of socks. I blindly draw socks from the dryer one at a time until I have a matching pair; let $X$ denote the number of socks taken from the dryer when this happens. Describe this experiment with a tree. Compute the PMF, mean, and variance of $X$.

(4) A student answers a True/False quiz with twenty questions by tossing a coin. What is the PMF, mean, and variance of the number of correct answers.