(1) Each of $n$ people are randomly and independently assigned a number from the set \{1, 2, 3, \ldots, 365\} according to the uniform distribution. We will call this number their birthday.
(a) What is the probability that no two people share a birthday?
(b) Use a computer or calculator to evaluate your answer as a decimal for $n = 22$ and $n = 23$.

(2) Suppose $X \sim \text{Poisson}(\lambda)$. By repeatedly differentiating the power series for $e^\lambda$, compute
$$
\mathbb{E}\{X(X-1)(X-2)\cdots(X+1-\ell)\}
$$
for each $\ell = 1, 2, 3, \ldots$.

(3) Suppose $X \sim \text{Binomial}(n, p)$. (a) By repeatedly differentiating the binomial identity compute
$$
\mathbb{E}\{X(X-1)(X-2)\cdots(X+1-\ell)\}
$$
for each $\ell = 1, 2, 3, \ldots$.
(b) If $n \to \infty$ with $p = \lambda/n$ show that your answer converges to the answer to the preceding question.

(4) Suppose $X \sim \text{Poisson}(\lambda)$. Compute the probability that $X$ is even.

(5) We deal seven cards from a well-shuffled standard deck of 52 cards. Compute the PMF, mean, and variance of the number of aces dealt.

(6) We throw a die independently four times and let $X$ denote the minimal value rolled.
(a) What is the probability that $X \geq 4$.
(b) Compute the PMF of $X$.
(c) Determine the mean and variance of $X$.

(7) Compute $\mathbb{E}(e^{tX})$ as a function of $t \in \mathbb{R}$ when
(a) $X \sim \text{Poisson}(\lambda)$,
(b) $X \sim \text{Binomial}(n, p)$.

(8) Problem 20 from Chapter 2.