(1) Determine all eigenvalues and eigenvectors of
\[
\begin{pmatrix}
1 & 0 \\
1 & 1
\end{pmatrix}
\].

(2) Find all eigenvectors of
\[
T : P(x) \mapsto \frac{1}{x} \int_{0}^{x} P(t) \, dt
\]
acting on the space of all polynomials with real coefficients of degree not exceeding one hundred.

(3) In this problem, we consider linear transformations of \( M_{2 \times 2}(\mathbb{R}) \)—the vector space of all \( 2 \times 2 \) matrices with real entries—back into itself. The standard basis consists of the four matrices that have 1 in one position and zeros everywhere else.

(a) Consider transposition: \( A \mapsto A^t \). Find the eigenvalues and eigenvectors.

(b) Let \( B = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \) and consider \( A \mapsto B^{-1}AB \). Write a matrix for this with respect to the standard basis. Find the eigenvalues and eigenvectors.

(4) We define a linear transformation, \( \text{tr} : M_{n \times n}(\mathbb{C}) \to \mathbb{C} \) by
\[
\text{tr}(A) = \sum_{k=1}^{n} A_{kk}
\].
Show that

(a) If \( A, B \in M_{n \times n}(\mathbb{C}) \) then \( \text{tr}(AB) = \text{tr}(BA) \).

(b) Similar matrices have the same trace.

(c) If \( A \) is diagonalizable, then \( \text{tr}(A) \) is the sum of the eigenvalues.