(1) Let $F = \mathbb{Z}_2$, the field with two elements. Find all bases for
\[ F^2 = \{ (x, y) : x, y \in F \}. \]
(2) Fix $v = (a, b, c)$ in $\mathbb{R}^3$ and define $T : \mathbb{R}^3 \to \mathbb{R}^3$ by
\[ T(x) = v \times x \]
where $\times$ denotes the cross product. Write a matrix for this with respect to the usual basis.
(3) Let $P = \{ a_0 + a_1 x + \cdots + a_4 x^4 : a_0, \ldots, a_4 \in \mathbb{R} \}$. Show that $T : P \to \mathbb{R}^2$ by
\[ T : P(x) \mapsto \begin{pmatrix} P(1) \\ P(2) \end{pmatrix} \]
is a linear transformation. Find a basis for its kernel.
(4) Let $P$ be as in the last question. Define $T : P \to P$ by $T : P(x) \mapsto P(x + 1)$, for example,
\[ T(x^3 + x) = (x + 1)^3 + (x + 1) = x^3 + 3x^2 + 4x + 2. \]
This is a linear transformation—you need not check.
(a) Write a matrix for $T$ in the basis $1, x, x^2, x^3, x^4$.
(b) Show that $T$ is onto.
(5) Study for the mid-term. You will be asked to define something and also to reproduce a proof from class. There will be other problems too.