Math 131AH Assignment 4

Due Tuesday, November 2, 2004


2. For $x \in \mathbb{Q}$ define the cut

$$\alpha_x = \{ z \in \mathbb{Q} : z < x \}.$$ 

Prove that if $x, y \in \mathbb{Q}$ then

$$\alpha_{x+y} = \alpha_x + \alpha_y$$

and

$$(\alpha_x)(\alpha_y) = \alpha_{(xy)}.$$ 

3. Prove that if $\alpha, \beta$ and $\gamma$ are cuts, then

$$\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma.$$ 

4. When $\alpha$ is a cut and $\alpha > 0^*$, define

$$1/\alpha = \{ x \in \mathbb{Q} : x \leq 0 \}$$

$$\cup\{ x \in \mathbb{Q} : x > 0, \text{ and there exists } z > x \text{ such that } y \in \alpha, y > 0 \Rightarrow zy < 1 \}.$$ 

Prove that $1/\alpha$ is a cut and $\alpha(1/\alpha) = 1^*$ where $1^* = \{ x \in \mathbb{Q} : x < 1 \}.$