Math 131AH Assignment 2

Due Tuesday, October 19, 2004

1. For $n$ and $m$ any two natural numbers, prove: $n \times 0 = 0$, $n \times (m^+) = n \times m + n$, and $n \times m = m \times n$.

2. For $a, b$ and $c$ arbitrary natural numbers, prove $a \times (b \times c) = (a \times b) \times c$.

3. For $a$ and $b$ arbitrary integers, prove

$$a \times b = 0 \Rightarrow a = 0 \text{ or } b = 0.$$ 

4. For $a, b$ and $c$ arbitrary integers, prove:
   (i) $a > b$ if and only if there is positive natural number $n$ such that $a = b + n$.
   (ii) If $a > b$, then $a + c > b + c$, and if $c$ is a positive natural number then $ac > bc$.
   (iii) $a > b$ if and only if $-b > -a$.
   (iv) If $a > b$ and $b > c$, then $a > c$.
   (v) If $a \geq b$ and $b \geq a$, then $a = b$.

5. Let $x$ be a rational number. Prove that exactly one of the following holds: (i) $x = 0$, (ii) $x$ is a positive rational number, or (iii) $x$ is a negative rational number.

6. Let $x, y$ and $z$ be rational numbers. Prove:
   (i) Exactly one of the three statements $x = y$, $x > y$, or $x < y$ holds.
   (ii) $x < y \iff y > x$.
   (iii) $(x < y \text{ and } y < z) \implies x < z$.
   (iv) $x < y \implies x + z < y + z$.
   (v) $(z \text{ positive and } x < y) \implies xz < yz$. 