Math 131AH Fall 2015: Homework 8, Due 11/20

1. Let \((X, d)\) be a metric space.
   
   (a) Show that \(X\) is connected if and only if every continuous function \(f : X \to \mathbb{N}\) is a constant.

   (b) Consider a continuous function \(g : X \to Y\), where \(Y\) is another metric space. Using (a), show that if \(X\) is connected, then \(g(X)\) is a connected subset of \(Y\). In particular, if \(X\) is an interval in \(\mathbb{R}\), then so is \(g(X)\).

2. Show that \(f : X \to Y\) is continuous on \(X\) if and only if \(f\) is continuous on every compact subset of \(X\).

3. Let \(f : [0, 1] \to [0, 1] \times [0, 1]\) be a continuous function and is onto. Show that \(f\) cannot be one-to-one on \([0, 1]\).

4-6. page 98, problem 1, 3, 18.