Math 131A-Practice Midterm 1.

Please write clearly, and show your reasoning with mathematical rigor. You may use any correct rule about the algebra or order structure of $\mathbb{R}$ from Section 3 without proving it.

1.
(a) State the Principle of Mathematical Induction.

(b) Prove that for all positive integers $n$,

$$1 + 3 + \ldots + (2n - 1) = n^2.$$ 

2.
(a) State the Least Upper Bound Axiom (also called the Completeness Axiom).

(b) Let $S$ be a non-empty subset of the real numbers such that $S$ is bounded above. Let $M = \{ y : y \geq x \text{ for all } x \in S. \}$. Prove that $M$ is non-empty, that $M$ is bounded below, and that

$$\sup S = \inf M.$$ 

3. Show that natural numbers do not have an upper bound (Do not use Archimedean Property).

4.
(a) Give the $\epsilon - N$ definition of $\lim_{n \to \infty} s_n = s$.

(b) Use the definition to prove that $\lim_{n \to \infty} \frac{10n + (-1)^n}{n} = 10$. 

1
5. Carefully prove that if \( s_n \to s \) and \( t_n \to t \), where \( s \) and \( t \) are real numbers, then

\[ s_n t_n \to st \text{ as } n \to \infty. \]

6.

(a) State the definition of \( \limsup_{n \to \infty} s_n \) and \( \liminf_{n \to \infty} s_n \) for a given sequence \( s_n \).

(b) Show that \( \liminf_{n \to \infty} s_n \leq \limsup_{n \to \infty} s_n \).