Deadline: Friday June 12 at 12:00 pm, noon.

Question 1. Show that a commutative ring $A$ is artinian if and only if it is noetherian and $\text{Spec}(A) = \text{Max}(A)$ (i.e. it has Krull dimension zero). Does left artinian imply left noetherian for non-commutative rings too?

Question 2. Show that a local noetherian domain $A$ of dimension one (i.e. $\text{Spec}(A) = \{0, m\}$ with $m \neq 0$) which is integrally closed is a DVR.

Question 3. Let $G$ be a finite group of exponent $m$. Let $K$ be a field whose characteristic does not divide the order of $G$ (for instance $\text{char}(K) = 0$). Is it enough for $K$ to contain a primitive $m$-th root of unity to be a splitting field of $G$?

Question 4. Let $G$ be a finite group whose character table contains the following two rows:

$\chi_1 : \begin{array}{cccc} 1 & 1 & 1 & \omega^2 & \omega & \omega^2 & \omega \\ \chi_2 : & 2 & -2 & 0 & -1 & -1 & 1 & 1 \end{array}$

where $\omega$ is a primitive cubic root of unity. Determine the rest of the character table. Give as much information on $G$ as you can.

Question 5. Since $S_3$ and $S_4$ are solvable, what is the general solution by radicals of a polynomial equation of degree three and four? (Give formulas.)