Homework No. 2

This homework should be submitted just before the beginning of class, on February 13th, 2012. Please write in a black ink pen, so it is clear and easy to read! Write your name in Capital letters on the top of each page and number the pages.

1. Let $A = \{x \in \mathbb{R}, x > 3 \text{ or } x < 2\}$, $B = \{x \in \mathbb{R}, x \leq 3 \text{ and } x > -1\}$, $C = \{x \in \mathbb{R}, x^2 > 4\}$, $D = \{x \in \mathbb{R}, x < 2 \text{ and } x^2 > 9\}$, $R = X$ (the universal set).
   (a) What are: $A^c$? $A \cup A^c$? $B^c$? $B \cup B^c$
   (b) What are: $A \cup B$? $A \cap B$?
   (c) Describe $C$ in a way that is easy to represent on a number line.
   (d) Sketch a representation of $C$ on a number line.
   (e) What is $C^c$?
   (f) Describe $D$ in a way that is easy to represent on a number line.
   (g) Sketch a representation of $D$ on a number line.
   (h) What are: $A \cap D$? $A \cup D$?

2. For what values of $x$, $x \in \mathbb{R}$, is the following (open) statement true?
   \[
   \frac{x^2 - 9}{x + 3} = x - 3
   \]
   Explain your answer.

3. Is the following statement true?
   “For every $y$, $y \in \mathbb{R}$, there exists an $x$, $x \in \mathbb{R}$, such that: $x^2 = y$”
   Explain your answer.

4. For what values of $x$ and $y$, $x, y \in \mathbb{R}$, is the following (open) statement true?
   \[x^2 = y\]
   Explain your answer.
5. Examine the following statement: "If \( n \) is an even number, then \( (n^2 - 3n + 1) \) is positive".
   (a) Give an example that contradicts the statement;
   (b) Give an example that does not contradict the statement;
   (c) Is the given statement true? Explain your answer.
   (d) What is the negation of this statement?
   (e) Is the negation true? Explain your answer.

6. Examine the following statement \( (x, y \in R) \): "If \( x < 2 \), then \( y > -3 \)."
   (a) What would be a counterexample to this statement?
   (b) What is the contrapositive of this statement?
   (c) What is the negation of this statement?
   (d) Is the negation true? Explain your answer.

7. Examine the following statement \( (x \in R) \): "If \( x^2 + 1 < 0 \), then \( -x^2 - 3 < 0 \)."
   (a) Is the premise true?
   (b) Is the conclusion true?
   (c) Is the statement true? Explain your answer.

8. Examine the following statement \( (x \in R) \): "If \( x^2 + 1 < 0 \), then \( -x^2 - 3 > 0 \)."
   (a) Is the premise true?
   (b) Is the conclusion true?
   (c) Is the statement true? Explain your answer.