Homework

This homework should be submitted just before the beginning of class, on February 6th, 2012. Please write in a black ink pen, so it is clear and easy to read!
Write your name in Capital letters on the top of each page and number the pages.

1. Let $A = \{1, 2, 5\}$ and $B = \{2, 7\}$. What are: $A \cup B$? $A \cap B$? $A \times B$?

2. Let $A = \{x \in N, \text{x is a multiple of 6}\}$, $B = \{x \in N, \text{x is a multiple of 15}\}$, and $N = X$ (the universal set). What are: $A^c$? $(A^c)^c$? $B^c$? $A \cup B$? $A \cap B$? $A^c \cup B^c$? $A^c \cap B^c$? $(A \cap B)^c$? $(A \cup B)^c$? $A^c \cup B^c$? $A^c \cap B^c$?

3. Let $A = \{x \in N, \text{x is a multiple of 6}\}$, $B = \{x \in N, \text{x is a multiple of 3}\}$, and $N = X$ (the universal set). What are: $A^c$? $(A^c)^c$? $B^c$? $A \cup B$? $A \cap B$? $A^c \cup B^c$? $A^c \cap B^c$? $(A \cap B)^c$? $(A \cup B)^c$? $A^c \cup B^c$? $A^c \cap B^c$?

4. Let $A = \{x \in N, \text{x is a multiple of 5}\}$, $B = \{x \in N, \text{x is a multiple of 3}\}$, and $N = X$ (the universal set). What are: $A^c$? $(A^c)^c$? $B^c$? $A \cup B$? $A \cap B$? $A^c \cup B^c$? $A^c \cap B^c$? $(A \cap B)^c$? $(A \cup B)^c$? $A^c \cup B^c$? $A^c \cap B^c$?

5. Try to prove that: $(A \cap B)^c = A^c \cup B^c$. Share your thinking on how to approach this problem.

6. Prove that: If $p$ and $q$ are two odd numbers, then $(p + q) \cdot (p - q)$ is a multiple of 4. Write a full proof in an acceptable form.

7. Assume that we know for sure that All Pelicans Eat Fish (i.e., this is a true statement).

(a) Which of the following statements follow from the above statement?

(b) Which of the following statements cannot be true, if the above statement is true?

   (i) If a bird is a Pelican, then it eats fish.

   (ii) If a creature eats fish, then it is a Pelican.

   (iii) If a bird is not a Pelican, then it does not eat fish.

   (iv) If a creature does not eat fish, then it is not a Pelican.

   (v) If a bird is a Pelican, then it does not eat fish.

   (vi) If a creature does not eat fish, then it is a Pelican.
8. Assume that we know for sure that: In a far away country named Shesing all woman who live there can sing (i.e., this is a true statement).

(a) Which of the following statements follow from the above statement?

(b) Which of the following statements cannot be true, if the above statement is true?

(i) If you live in Shesing and are a woman, then you can sing.
(ii) If you live in Shesing and can sing, then you are a woman.
(iii) If you live in Shesing and are not a woman, then you cannot sing.
(iv) If you live in Shesing and cannot sing, then you are not a woman.
(v) If you live in Shesing and are a woman, then you cannot sing.
(vi) If you live in Shesing and cannot sing, then you are a woman.

9. For each of the following statements determine whether is always true, sometimes true, or never true:

(i) When you add two rational numbers, you get the same result as when you multiply them.
(ii) \((a+b)^2 = a^2 + b^2\), when \(a, b \in \mathbb{R}\).