Please provide complete and well-written solutions to the following exercises.

Due January 12th, in the discussion section.

**Homework 1**

**Exercise 1.** Let $n$ be a positive integer. Consider the game Chomp played on an $n \times n$ board. Explicitly describe the winning strategy for the first player. (Hint: the first move should remove the square which is diagonally adjacent to the lower left corner.)

**Exercise 2.** Compute the following nim-sums: $3 \oplus 4$, $5 \oplus 9$. Then, let $a, b, c$ be nonnegative integers. Prove that $a \oplus a = 0$ and $(a \oplus b) \oplus 0 = a \oplus b$.

**Exercise 3.** Consider the nim position $(9, 10, 11, 12)$. Which player has a winning strategy from this position, the next player or the previous player? Describe a winning first move.

**Exercise 4.** Consider the game of Chomp played on a board of size $2 \times \infty$. Recall that a typical Chomp game board is $n \times m$, so that the board has $n$ rows and $m$ columns. We can label the rows as $\{1, 2, \ldots, n\}$ and we can label the columns as $\{1, 2, \ldots, m\}$, where $n, m$ are positive integers. On a $2 \times \infty$ board, we label the rows as $\{1, 2\}$, and we label the columns as $\{1, 2, 3, 4, 5, 6, \ldots\}$. We can think of the row and column labels as coordinates in the $xy$-plane. So, the lower left corner will still have $x$-coordinate 1 and $y$-coordinate 1, so that the lower left square has coordinates $(1, 1)$; the square to the right of this has coordinates $(2, 1)$, and so on.

On the $2 \times \infty$ board, which player has a winning strategy? Prove your assertion, and describe explicitly the winning strategy.

Let $n > 2$ be an integer. On the $n \times \infty$ board, which player has a winning strategy? Prove your assertion, and describe explicitly the winning strategy.

On the $\infty \times \infty$ board, which player has a winning strategy? Prove your assertion, and describe explicitly the winning strategy.

**Exercise 5.** Let $G_1, G_2$ be games. Let $x_i$ be a game position for $G_i$, and let $N_{G_i}, P_{G_i}$ denote, $N$ and $P$ respectively for the game $G_i$, for each $i \in \{1, 2\}$. Show the following:

(i) If $x_1 \in P_{G_1}$ and if $x_2 \in P_{G_2}$, then $(x_1, x_2) \in P_{G_1+G_2}$.

(ii) If $x_1 \in P_{G_1}$ and if $x_2 \in N_{G_2}$, then $(x_1, x_2) \in N_{G_1+G_2}$.

(iii) If $x_1 \in N_{G_1}$ and if $x_2 \in N_{G_2}$, then $(x_1, x_2)$ could be in either $N_{G_1+G_2}$ or $P_{G_1+G_2}$.

**Exercise 6.** Let $G_1, G_2, G_3$ be games. Show that the notion of two games being equivalent is an equivalence relation. That is, show the following

- $G_1$ is equivalent to $G_1$.
- If $G_1$ is equivalent to $G_2$, then $G_2$ is equivalent to $G_1$. 
If $G_1$ is equivalent to $G_2$, and if $G_2$ is equivalent to $G_3$, then $G_1$ is equivalent to $G_3$. 