Math 223d, HW1

Due date: Wednesday, April 14, start of class (11:30am).

Q1: Let $T \subseteq A^{<\mathbb{N}}$ be an infinite tree with finite branching (i.e. every $s \in T$ has only finitely many $a \in A$ with $s \mathbin{\rhd} a \in T$). Show that $T$ has an infinite branch. ("König’s infinity lemma").

Q2: Let $T \subseteq A^{<\mathbb{N}}$ be a pruned non-empty tree. Equip $[T]$ with the topology generated by basic open sets of the form

$$N_s = \{x \in [T] : x \supset s\}$$

for $s \in T$.

Show that $[T]$ is separable (i.e. has a countable dense subset) if and only if $T$ is countable.