1. (10 points) Suppose that a swan is swimming in a circle described by:

\[ c(t) = (\cos(t), \sin(t)), \quad t \in \mathbb{R}^+ \]

and the water temperature is given by:

\[ T(x, y) = x^2e^y, \quad x, y \in \mathbb{R} \]

(a) Find \( DT(xy) \) and explain what this represents.

(b) Find \( D(T \circ c)(t) \) and explain what this represents.

2. (10 points) Determine whether the following statements are true or false. If the statement is true, indicate how you could show it. If the statement is false, provide a counterexample.

(a) If \( f(x, y, z) = x^4 + xy + z^3 \), then \( \nabla f(1, 0, 1) \) is perpendicular to the surface \( f = 2 \) at the point \( (1, 0, 1) \).

This statement is (circle one): \( \text{true} \quad \text{false} \)

Explaination or counterexample:

(b) If \( f(x, y) = \ln y \), then \( \nabla f(x, y) = \frac{1}{y} \)

This statement is (circle one): \( \text{true} \quad \text{false} \)

Explaination or counterexample:

3. (10 points) Find the absolute maxima and minima of the function

\[ f(x, y) = 5x^2 - 2y^2 + 2 \]

on the disk \( x^2 + y^2 \leq 1 \).

4. (10 points) Pictured are a contour map of \( f \) and a (dashed) curve with equation \( g(x, y) = 8 \). Estimate the maximum and minimum values of \( f \) subject to the constraint \( g(x, y) = 8 \). Explain your choices.