1. Let $S$ be any axiomatic system whose axioms include all tautologies and whose rules of inference include MP. Prove that $S$ is sound if and only if every satisfiable set of formulas is consistent in $S$.

   Hint. If $\Phi \vdash_S A$, then $\Phi \cup \{\neg A\} \vdash_S B$ for every formula $B$. (Explain why.)

2. Do the $\neg$ and $\supset$ cases of the proof of Lemma 1.20.