Example:
\[ \begin{align*}
\text{1.a)} \\
&\frac{d}{dt} y(t) = y - \sin(t) \\
&f(t,y) = y - \sin(t) \\
&f(t,y_1) - f(t,y_2) = |y_1 - \sin(t) - y_2 - \sin(t)| = |y_1 - y_2| \\
&\leq 1 \cdot |y_1 - y_2| \\
\end{align*} \]

\[ \begin{align*}
\text{1.b)} \\
&y' = \frac{2}{t} y + t^2 \quad 1 \leq t < 2; \quad y(1) = 0 \\
&|f(t,y_1) - f(t,y_2)| = \left| \frac{2}{t} y_1 - \frac{2}{t} y_2 \right| \\
&= \frac{2}{t} |y_1 - y_2| \\
&\leq 2 |y_1 - y_2| \\
\end{align*} \]

\[ \begin{align*}
\text{1.c)} \\
&y' = \frac{4t^3}{1 + t^4} \\
&f(t,y_1) - f(t,y_2) = \left| \frac{4t^3 y_1^2}{1 + t^4} - \frac{4t^3 y_2^2}{1 + t^4} \right| \\
&= \frac{4t^3}{1 + t^4} |y_1 - y_2| \leq 2 |y_1 - y_2| \\
\end{align*} \]

\[ \begin{align*}
\text{1.d)} \\
&y' = 9y \\
&0 \leq t < 1 \\
&f(t,y_1) - f(t,y_2) = |9y_1 - 9y_2| \leq 9 |y_1 - y_2| \\
\text{Lipschitz con.} \\
\end{align*} \]
Proof Theorem 5.10.

This is analogous to Theorem 5.9.

\[ |Y_{i+1} - w_{i+1}| = |Y_i - w_i| + \frac{1}{\epsilon} \frac{h^2}{\phi} (f_i(w_i) - f_i(Y_i)) + \frac{1}{\epsilon} \frac{h^2}{\phi} Y_i (S) \]

Since all \( |f_i| \leq p \),

\[ |Y_{i+1} - w_{i+1}| \leq (1 + 2h) |Y_i - w_i| + \frac{h^2}{\epsilon} + p \]

\[ \text{Set } \lambda = \frac{1}{\epsilon} \frac{h^2}{\phi} \]

\[ |Y_{i+1} - w_{i+1}| \leq \lambda \left[ |Y_0 - w_0| + \frac{h^2}{\phi} m + \frac{p}{\phi} \right] + \frac{h^2}{\phi} \frac{\lambda}{\phi} \]

\[ \leq (1 + \lambda h) \left[ |Y_0 - w_0| + \frac{h^2}{\phi} m + \frac{p}{\phi} \right] \]

\[ \leq \frac{1}{\epsilon} \left( \frac{h^2}{\phi} + \frac{p}{\phi} \right) + p \]

\[ |Y_i - w_i| \leq \frac{1}{\epsilon} \left( \frac{h^2}{\phi} + \frac{p}{\phi} \right) \left[ e^{\lambda h} - 1 \right] + \frac{1}{\phi} \frac{h^2}{\phi} \lambda \frac{\epsilon h}{\phi} \]

\[ Q.E.D. \]