1. Using induction and the basic trigonometric identities for \( \sin(x + y) \) and \( \cos(x + y) \), prove de Moivre’s identity:

\[
(\cos(z) - i\sin(z))^n = \cos(nz) - i\sin(nz)
\]

where \( i^2 = -1 \).

2. Let \( e = \sum_{n=0}^{\infty} 1/n! \). Estimate the error term \( e - \sum_{n=0}^{k} 1/n! \). (Hint: Compare the infinite "tail" of the sum to the whole series.)

3. Use Euler’s expression \( e^x = (1 + x/N)^N \) for an infinitely large integer \( N \) to show that \( f(x) = e^x \) is equal to its own derivative. (Hint: use this to compute \( e^{dx} \) for an infinitesimal \( dx \) by applying binomial expansion and dropping higher-order differentials.)

4. Let \( f \) be any differentiable function of one variable. Show that the function of two variables

\[
y(x, t) = \frac{1}{2} f(x + at) + \frac{1}{2} f(x - at)
\]

satisfies the partial differential equation

\[
\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}.
\]