1. Show that

\[ \int_0^1 \frac{1 + x^2}{1 + x^4} \, dx = 1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \ldots \]

by expanding in a series and integrating termwise. (Can you find a closed expression for all the terms of the series?) (5 pts)

2. Suppose (using Newton’s fluxion notation) that \( \frac{\dot{y}}{\dot{x}} = \left( \frac{c}{x} \right) \sqrt{ax^n + bx^{2n}} \). Use substitution and termwise integration of power series to find an expression for the integral \( y \). (5 pts)

3. Given the equation \( y^3 + y - xy - x^3 - 2 = 0 \) find the first three terms of a power series expression of \( y \) as a function of \( x \). (5 pts)

4. Apply Newton’s method of successive approximation to invert the series

\[ \theta = \sin^{-1}(x) = x - \frac{1}{6} x^3 - \frac{1}{40} x^5 - \frac{1}{112} x^7 + \ldots \]

to get a series for \( \sin(\theta) \), up to degree 7. (5 pts)