**Problem 1.** Do Exercise 1, Section 2.1. Justify each answer!

**Problem 2.** Suppose $T : V \to W$ is a linear transformation of vector spaces. Let $V'$ be a subspace of $V$, and let $W'$ be a subspace of $W$.

1. Prove that $T(V')$ is a subspace of $W$.
2. Prove that $\{v \in V : T(v) \in W'\}$ is a subspace of $V$.

**Problem 3.** Let $V$ be a linear transformation. For each of the following linear transformations, compute $\gamma(T)$.

1. Give an example of distinct linear transformation $T : V \to W$ such that $N(T) = R(T)$.
2. Give an example of two distinct linear transformations $T$ and $U$ such that $N(T) = N(U)$ and $R(T) = R(U)$.

**Problem 4.** Let $V, W$ be vector spaces, and suppose $T : V \to W$ is a linear transformation. Suppose moreover that $T$ is bijective.

1. Prove that if $\{v_1, \ldots, v_n\}$ is a basis for $V$, then $\{T(v_1), \ldots, T(v_n)\}$ is a basis for $W$.

**Problem 5.**

1. Give an example of distinct linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $N(T) = R(T)$.
2. Give an example of two distinct linear transformations $T$ and $U$ such that $N(T) = N(U)$ and $R(T) = R(U)$.

**Problem 6.** For each of the following linear transformations, compute $[T]_\beta^\gamma$.

1. $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(a_1, a_2) = (2a_1 - a_2, 3a_1 + 4a_2, a_1)$, for $\beta$ and $\gamma$ the standard bases in $\mathbb{R}^2$ and $\mathbb{R}^3$, respectively.
2. $T : \mathbb{R}^n \to \mathbb{R}^n$ defined by $T(a_1, \ldots, a_n) = (a_n, a_{n-1}, \ldots, a_1)$, for $\beta = \gamma$ the standard basis in $\mathbb{R}^n$.
3. $T : M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ defined by $T(A) = A'$ for $\beta = \gamma$ the standard basis in $M_{2\times 2}(\mathbb{R})$ (i.e. $\beta = \gamma = \{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\}$).
4. $T : P_2(\mathbb{R}) \to \mathbb{R}$ defined by $T(f(x)) = f(2)$ for the standard bases $\beta = \{1, x, x^2\}$ and $\gamma = \{1\}$.
5. $T : M_{2\times 2}(\mathbb{R}) \to P_2(\mathbb{R})$ defined by $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a + b) + (2d)x + bx^2$, for $\beta$ the standard basis of $M_{2\times 2}(\mathbb{R})$ and $\gamma$ the standard basis of $P_2(\mathbb{R})$.

**Problem 7.** Let $V, W$ be vector spaces, and let $T, U$ be non-zero linear transformations from $V$ to $W$. Prove that if $R(T) \cap R(U) = \{0\}$, then $T$ and $U$ are linearly independent vectors in $\mathcal{L}(V, W)$.

**Problem 8.** Prove Theorem 2.10.