1. Solve the initial value problem
\[
x u_x - y u_y = \cos(y) \\
u(x, 0) = x.
\]
2. Find the characteristic equations for the following PDE:
\[
u^2_x + u^4_y = x^2 u^2.
\]
3. Solve the initial value problem
\[
u_{tt} = c^2 u_{xx} \\
u(x, 0) = e^x \\
u_t(x, 0) = e^{2x}.
\]
4. For \(at \leq x\) and \(t \geq 0\), and for some constant \(a\), consider
\[
u_{tt} = c^2 u_{xx} \\
u(x, 0) = g(x) \\
u_t(x, 0) = h(x) \\
u(x = at, t) = k(t).
\]
- Show that this is well-posed if \(|a| < c\) but ill-posed if \(|a| > c\).
- For \(|a| < c\), find the solution of this problem.
5. Consider the initial value problem
\[
u_{tt} = c^2 \Delta u
\]
for \(x \in \mathbb{R}^d\) and \(t > 0\), and with \(u(x, 0) = u_0(x)\) \(u_t(x, 0) = u_1(x)\) in which \(u_0(x) = u_1(x) = 0\) for \(|x| < R_1\) and \(|x| > R_2\), with \(R_1 < R_2\). For \(d = 2\) and \(d = 3\), find the largest set \(\Omega_0 \subset \{x \in \mathbb{R}^d, t > 0\}\) on which \(u = 0\) for any choice of \(u_0\) and \(u_1\).