HW#2: due Fri 10/31/2014

**Problem 1:** Show that, for a sequence of sets \( \{A_n\} \) and a set \( A \), we have \( \lim_{n \to \infty} A_n = A \) if and only if \( 1_{A_n} \to 1_A \) pointwise. Here \( 1_A \) is the characteristic function of \( A \).

**Problem 2:** Ex 1.2.15, Page 33

**Problem 3:** Ex 1.2.17, Page 33

**Problem 4:** Ex 1.2.18, Page 33

**Problem 5:** Ex 1.2.20, Page 34

**Problem 6:** Ex 1.2.21, Page 34

**Problem 7:** Ex 1.2.23, Page 35

**Problem 8:** Show that any \( \sigma \)-algebra of sets is either finite or uncountable.

**Problem 9:** Ex 1.2.24, Page 35

**Problem 10:** Let \( \Omega \) be any set. Prove that if \( \{A_\alpha : \alpha \in \mathcal{I}\} \) is any collection of \( \sigma \)-algebras of subsets of \( \Omega \), then their intersection, \( \bigcap_{\alpha \in \mathcal{I}} A_\alpha \), is also a \( \sigma \)-algebra. Check that both \( \{\emptyset, \Omega\} \) and the power set \( 2^\Omega := \{A: A \subset \Omega\} \) are \( \sigma \)-algebras.

**Problem 11:** Let \( \mu \) be a finite measure on any \( \sigma \)-algebra \( \mathcal{A} \). Assume that \( \mu \) is *non-atomic* in the sense that for each \( A \in \mathcal{A} \) with \( \mu(A) > 0 \) there is a \( B \in \mathcal{A} \) with \( B \subset A \) such that \( 0 < \mu(B) < \mu(A) \). Prove that \( \{\mu(A): A \in \mathcal{A}\} \) is a closed interval containing 0.

**Problem 12:** Ex 1.2.27, Page 38

**Problem 13:** Ex 1.3.3, Page 50

**Problem 14:** Ex 1.3.4, Page 50

**Problem 15:** Ex 1.3.5, Page 50

**Problem 16:** Ex 1.3.6, Page 50