Assignment #9

Note quiz announcement below.

Homework assignment #9 is due in lecture Friday, June 2. Be sure to try these problems before your discussion section!

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Quiz in section, Week 9 (May 30 and June 1): Know:
(a) formulas for $\cosh x$, $\cos(x + y)$, $\cos(x - y)$, $\sin(x + y)$, $\sin(x - y)$;
(b) formula for $\cos(mx) \cos(nx)$ and how to derive it;
(c) $\int_{-\infty}^{\infty} \cos(mx) \cos(nx) \, dx$ (for $m, n \geq 0$, cases $m = n$ and $m \neq n$);
(d) formulas for the Fourier coefficients $a_n, b_n$ of a periodic function $f$ of period $2\pi$;
(e) the reasoning showing how the formulas in (d) are derived.

Problem Q-1. Let $S$ be any set of real numbers bounded above. As you know, an upper bound for $S$ is any $M$ so that $s \leq M$ for all $s$ in $S$. It is a fact that $S$ always has a least upper bound, called the supremum of $S$, or sup $S$. If $S$ is not bounded above, we can write sup $S = \infty$. If $S$ has a maximum element (i.e., largest number), then that is sup $S$. Find sup $S$ in the following cases.
(a) $S$ is the interval $(-\infty, 2]$.
(b) $S$ is the set of all numbers $3 - \frac{1}{n}$, $n = 1, 2, 3, \ldots$.
(c) $S$ is the set of all numbers $3 + \frac{1}{n}$, $n = 1, 2, 3, \ldots$.
(d) $S$ is the set of all integers (whole numbers).
(e) $S$ is the set of all values of $\frac{x}{x+1}$ for $x \geq 1$. 

Q 1
Problem Q-2. The “sup norm” of a function $f$ on an interval $I$ is the sup of the set of values of $|f|$ on $I$. We denote the sup of $f$ on $I$ by $\|f\|$. (Often people write $\|f\|_\infty$, but we won’t.) If $f$ has a maximum absolute value on $I$, then that’s the value of $\|f\|$. Find $\|f\|$ in these cases:
(a) $f(x) = xe^{-x}$ on $(0, \infty)$.
(b) $f(x) = x^2$ on $[-4, 3]$.
(c) $f(x) = x^2$ on $[0, 2)$.
(d) $f(x) = 1/x$ on $(0, \infty)$.
As you see, this concept is handy because $\|f\|$ always has a value even if there is no actual maximum value. Remember, though, that a continuous function on a closed interval $[a, b]$ does have a maximum value (so that’s its sup norm).

Problem Q-3. For functions $f, g$ on an interval, $\|f - g\|$ is essentially the maximum distance between the two functions. Draw a sketch showing two continuous functions $f, g$ on $[0, 2]$ with $\|f - g\| < \frac{1}{4}$, with $f$ above $g$ in some places and below in others.

Problem Q-4. The sup norm gives an easy way of thinking about uniform convergence: $f_n \to f$ on an interval $I$ means that $\|f_n - f\| \to 0$ as a sequence of numbers. That’s all! (Let’s stay away from cases where the value would be infinite; in such a case we’d have to say that the value of $\|f_n - f\|$ is eventually finite). In any problem on uniform convergence you can write $\|f_n - f\|$ instead of $M_n$ or $\epsilon_n$.

Do p. 163, Problem 5 using this notation.