Assignment #5

No quiz during Week 5, because of the midterm.
This short assignment is due in lecture Friday, May 5. Be sure to try these
problems before your discussion section, even if you have to get some infor-
mation from reading the text.

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Problem J-1. As mentioned in lecture, if you have some game or ex-
periment where you have different probabilities of getting different scores, the
“expected value” is the sum of the scores weighted by the probabilities—a
weighted average. Examples:

1. Suppose you have probability \( \frac{1}{6} \) of getting 7, probability \( \frac{1}{3} \) of getting 13, and probability \( \frac{1}{2} \) of getting 2. (That’s all, since \( \frac{1}{6} + \frac{1}{3} + \frac{1}{2} = 1 \).)
   Then the expected value is \( \frac{1}{6} \cdot 7 + \frac{1}{3} \cdot 13 + \frac{1}{2} \cdot 2 = \frac{39}{6} = 6.5 \).

2. In a lottery, if you have a chance of one in a million of winning $2
   million (and otherwise you will win nothing), then your expected value is
   \( 0.000001 \cdot 2000000 + 0.999999 \cdot 0 = 2 \) (so you’d better not have to
   pay much more than that for a ticket).

3. If you throw a die repeatedly until you get a 5, the expected number of
   throws is \( \frac{1}{6} \cdot 1 + \frac{5}{6} \cdot \frac{1}{6} \cdot 2 + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} \cdot 3 + \cdots = \sum_{n=1}^{\infty} (\frac{5}{6})^{n-1} \frac{1}{6} n \), because your
   chance of needing one throw is \( \frac{1}{6} \), your chance of needing two throws is
   \( \frac{5}{6} \cdot \frac{1}{6} \) (one failure followed by one success), and so on. In this example
   there are infinitely many possibilities, but the same principle applies.

The problem:
(a) In Example 3 above, find an explicit value for the expected number of
   throws, by adding up the infinite series. (Take the \( \frac{1}{6} \) outside and put \( x \) for \( \frac{5}{6} \)
temporarily so it’s easier to work with. Is your answer reasonable?)
(b) Suppose you do some other activity repeatedly until you succeed, and
   suppose the chance of success on each try is \( \frac{1}{N} \). Evaluate the same kind of
   sum to find the expected number of tries, in terms of \( N \).
(c) Although we won’t go into it, in Example 3 the sum \( \sum_{n=1}^{\infty} (\frac{5}{6})^{n-1} \frac{1}{6} n^2 \) is
   also useful. Find its value.