In this exercise you show that if $f$ is a twice continuously differentiable function of two variables\(^1\) then the Jacobian of the Gradient of $f$ is the Hessian of $f$.

**AP 1. If**

$$f : U \subset \mathbb{R}^2 \rightarrow \mathbb{R} \\
(x_1, x_2) \mapsto f(x_1, x_2)$$

is such that $f \in C^2(U)$, show that

$$\nabla(\nabla f(x))^T = \nabla^2 f(x).$$

Note that we have the transpose on the left hand side, yet there is no transpose on the right hand side. Do not forget to argue why this is the case.

\(^1\)This is also true if $f$ is a twice continuously differentiable function of $n$ variables, $n \in \mathbb{N}$. 