1. Prove that \( \text{ed}(\mathcal{A} \times \mathcal{B}) \leq \text{ed}(\mathcal{A}) + \text{ed}(\mathcal{B}) \).
2. Prove that the functor \( K \mapsto \text{M}_n(K)/(\text{similarity relation}) \) is represented by an affine scheme.
3. Let \( L/F \) be a finite field extension and \( G \) is the group of all automorphisms of \( L \) over \( F \). Prove that \( \text{Spec}(L) \) is a \( G \)-PHS if and only if \( L/F \) is a Galois field extension.
4. Prove that every \( \mu_n \)-PHS is of the form \( \text{Spec}(F[t]/(t^n - a)) \) for some \( a \in F^\times \).
5. Prove that a twisted form of the matrix algebra \( \text{M}_n(F) \) is a central simple \( F \)-algebra of degree \( n \).
6. Prove that a twisted form of the algebra \( F_n := F \otimes F \) is a product of finitely many finite separable field extensions of \( F \).
7. Let \( X \rightarrow Y \) be a \( G \)-torsor. Prove that the morphism \( G \times X \rightarrow X \times_Y X \) taking \( (g, x) \) to \( (gx, x) \) is an isomorphism.
8. Determine the essential dimension of the alternating group \( A_4 \).
9. Let \( G \) be a finite group acting faithfully of a finite dimensional vector space \( V \). Suppose every eigenspace of every nontrivial element of \( G \) is different from \( V \). Prove that \( \text{ed}(G) \leq \text{dim}(V) - 1 \).
10. Determine the essential dimension of the functor \( K \mapsto \text{the set of isomorphism classes of non-degenerate quadratic forms of dimension } n \text{ over } K \text{ with trivial discriminant} \).
11. Prove that \( \text{cdim}(X) = \text{cdim}(X \times X) \) for every variety \( X \).
12. Let \( A \) and \( B \) be two central simple \( F \)-algebras. Prove that \( SB(A) \times SB(B) \) is isomorphic to a closed subvariety of \( SB(A \otimes_F B) \).
13. Let \( A \) be a central simple \( F \)-algebra. Consider the category \( \mathcal{X} \) whose objects over a scheme \( X \) are the pairs \( (E, \varphi) \) such that \( E \) is a vector bundle over \( X \) and \( \varphi : A \otimes O_X \rightarrow \text{End}(E) \) is an algebra isomorphism. A morphism between pairs is given by a morphism between vector bundles commuting with \( \varphi \). Prove that \( \mathcal{X} \) is a gerbe banded by \( \mathbb{G}_m \).
14. Let \( V \) be a vector space over \( \mathbb{F}_p \) of dimension \( n \), \( p \) a prime, and let \( f : V \setminus \{0\} \rightarrow \mathbb{R} \) be a function. Define inductively the vectors \( v_1, v_2, \ldots, v_n \) as follows: \( v_i \) is a vector in \( V \setminus \text{Span}(v_1, \ldots, v_{i-1}) \) with the smallest value \( f(v_i) \). Prove that the basis \( B := \{v_1, \ldots, v_n\} \) is minimal, i.e., the sum

\[ \sum_{v \in B} f(v) \]

is the smallest over all bases \( B \).