1. Determine whether the subset \{(1, 2), (2, 1)\} is basis for \(\mathbb{R}^2\).

2. The vectors \(u_1 = (2, -3, 1), u_2 = (1, 4, -2), u_3 = (-8, 12, -4), u_4 = (1, 37, -17), u_5 = (-3, -5, 8)\) generate \(\mathbb{R}^3\). Find a subset of the set \(\{u_1, u_2, u_3, u_4, u_5\}\) that is a basis for \(\mathbb{R}^3\).

3. Let \(\{u, v\}\) be a basis for \(V\). Show that \(\{2u + 3v, u + 2v\}\) is also basis.

4. Let \(W_1\) and \(W_2\) be two subspaces of a vector space \(V\). Show that \(\dim(W_1 \cap W_2) = \dim(W_1)\) if and only if \(W_1 \subset W_2\).

5. Find the dimension of the spaces \(\text{Sym}_{n \times n}(\mathbb{R})\) and \(\text{Skew}_{n \times n}(\mathbb{R})\) of symmetric and skew-symmetric \(n \times n\) matrices respectively.

6. Prove that the subset of \(F^n\) consisting of all vectors \((a_1, a_2, \ldots, a_n)\) such that \(a_1 + a_2 + \ldots + a_n = 0\) is a subspace of \(F^n\) and find its dimension.

7. Prove that the subset of \(P_n(F)\) consisting of all polynomials \(f\) such that \(f(1) = 0\) is a subspace of \(P_n(F)\) and find its dimension.

8. Let \(V\) be a finite dimensional vector space and \(S \subset V\) a subset (possibly infinite) with \(\text{span}(S) = V\). Prove that some subset of \(S\) is a basis for \(V\).

9. Let \(W_1\) and \(W_2\) be finite dimensional subspaces of a vector space \(V\). Prove that the subspaces \(W_1 \cap W_2\) and \(W_1 + W_2\) are also finite dimensional and

\[
\dim(W_1 \cap W_2) + \dim(W_1 + W_2) = \dim(W_1) + \dim(W_2).
\]

10. (*) Let \(V\) be a vector space of dimension \(n\) and let \(V_1, V_2, \ldots, V_k \subset V\) be subspaces of \(V\). Assume that

\[
\sum_{i=1}^{k} \dim(V_i) > n(k - 1).
\]

Prove that \(\bigcap_{i=1}^{k} V_i \neq \{0\}\).