Write your solution on the exam sheet. Show some work and justify your answer. Do not just give the correct answer. You have 15 minutes. Good luck!

1. Let

\[ F(x, y, z) = x^2 + 2y^2 + 3z^2. \]

At the point (2, 1, 1) find the direction \( \vec{u} = < u_1, u_2, u_3 > \) (with \( u_1^2 + u_2^2 + u_3^2 = 1 \)) in which \( F \) has the greatest increase.

Solution: Since the the directional derivative \( D_u F = \nabla F \cdot \vec{u} \), the direction of greatest increase for \( F \) is always

\[ \vec{u} = \frac{\nabla F}{|\nabla F|}. \]

In our case we have

\[ \nabla F = < 2x, 4y, 6z > = < 4, 4, 6 > \]

and

\[ \vec{u} = \frac{1}{\sqrt{17}} < 2, 2, 3 > . \]