Letting $y$ vary for $x=y$, we can obtain any nonnegative real number. Then the range is $[0, \infty)$.

17. Domain: We must have $y-x^2 \geq 0$ so that the square root is defined. Also, $1-x \neq 0$ (denominator), $y-x^2 \neq 0 \Rightarrow y \neq x^2$.

Range: Let $x=0$. Then varying $y$ we obtain any nonnegative real number: $\sqrt{y-x^2} = y-x^2 \geq 0$. Now let $x=2$: $\sqrt{y-4} = y-4 \geq 0$.

8. Domain: $1+x-y \geq 0 \Rightarrow y \leq 1+x$.

Range: Some as the range of $y = \ln(x+y-1) = \ln(x+y-1)$.

15. $f(x,y) = \ln(x+y-1)$
   (a) $f(x,0) = \ln(x+0-1) = \ln(x-1)$
   (b) $f(0,y) = \ln(0+y-1) = \ln(y-1)$
   (c) Domain: we need $x+y-1 > 0 \Rightarrow y > x-1$.
Cross sections of a paraboloid.

Consider the family of paraboloids

\[ z = x^2 + y^2 \]

We get a paraboloid. The cross section is a circle.

\[ x^2 + y^2 = r^2 \]

Let \( x + y = r \) and consider the plane \( x + y = c \).

We have a maximum at \( (x, y) = (0, 0) \) because of the \( x^2 + y^2 \) term.

We have a minimum at \( (x, y) = (0, 0) \) and then \( f(x, y) = 0 \).

If \( x = 0 \) or \( y = 0 \), \( f(x, y) = 0 \).

If \( x = y \), then \( f(x, y) = 0 \).

We should have \( x = y \) so the straight lines of slope 1.

Consider lines of constant \( x - y = \text{constant} \).

All values of \( x \).

You can think of it as taking the line \( \lambda y = x \). A plane where \( y = \lambda \).

The set of points inside an ellipse.

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} < 1 \]

So we must have \( 16 - 4x^2 - 4y^2 - 2y > 0 \).
the y direction.

So we have the same as in part a, but shifted 1 units in

\[ x^2 + (y+1)^2 - 1 = 0 \]

(c) We have \( x^2 + y^2 + 2y - 2 = x^2 + (y+1)^2 - 1 \), so the graph should be the same with \( y \) flipped, so we still have a hyperbola on one sheet.

So this is a hyperbola and rotating it around the y-axis.

For \( z = c \), \( x^2+y^2 = 1+c \), circles.

Similarly, for \( y = c \).

9. (a) For \( x = c \) (constant), \( y^2 - x^2 = 1-c^2 \), so the traces are hyperbolas.

\[ \text{One piece:} \]

6. \( \dot{y}(t) \) gives a hyperbola in the \( y-t \) plane. This can vary over all \( y \).

13.6

Only III has periodicity. For part III, \( f'(0) = \text{sin} \), which is periodic.

30. (f) We get \( f'(0) \) = sin, which is periodic.
22. Dttb, we get \( \frac{x^2}{1^2} + \frac{z^2}{1^2} = 1 \). This is the basic equation of an ellipsoid slightly disguised.

23. From \#19, this is a hyperboloid of one sheet.

24. This is the basic form of a hyperboloid of two sheets.

25. Paraboloid opening up toward \( y \) axis.

26. Cones opening up toward \( y \) axis.

27. This is an ellipse in the \( xy \)-plane. We can vary \( y \) to get a cylinder-like figure.

28. For \( z = 0 \), we get \( y = x^2 \), so the projection in the \( xy \)-plane is a parabola.

29. Complete the square: \((x-1)^2 - (y-1)^2 + (z+2)^2 = 2 \Rightarrow \frac{(x-1)^2}{\sqrt{2}} - \frac{(y-1)^2}{\sqrt{2}} + \frac{(z+2)^2}{\sqrt{2}} = 1\) the standard form for the hyperboloid of one sheet.
15.2

7. Let \( y = mx \). Then

\[
\frac{x^2}{x^2 + mx^2} = \frac{x^2}{x^2 (1 + m^2)}.
\]

12. Let \( y = mx \). Then

\[
\frac{2x + y}{\sqrt{x^2 + y^2}} = \frac{2x + mx}{\sqrt{x^2 + (mx)^2}} = \lim_{m \to 0} \frac{2x + mx}{\sqrt{x^2 (1 + m^2)}} = \lim_{m \to 0} \frac{2x + mx}{x \sqrt{1 + m^2}} = \frac{2}{\sqrt{1 + 0^2}} = \frac{2}{1} = 2.
\]

13. Let \( y = mx \). Then

\[
\frac{x^2}{x^2 + mx^2} = \frac{x^2}{x^2 (1 + m^2)}.
\]

14. The distance from \((x, y)\) to \((-1, 0)\) is

\[
\sqrt{(x + 1)^2 + y^2} = \sqrt{(x - (-1))^2 + y^2}.
\]

So the distance \((x - 1, y)\) to the plane \(x = 1\) is just the distance from \((x, y)\) to \((1, y)\).

Now if the two distances are equal,

\[
x - 1 = \sqrt{(x + 1)^2 + y^2} \Rightarrow x^2 + 2x + 1 = x^2 + y^2 + 2x + y^2 \Rightarrow y^2 = 0 \Rightarrow y = 0.
\]

And the distance from \((x, y)\) to \((-1, 0)\) is \(\sqrt{(x - (-1))^2 + y^2}\), the line connecting these two points is perpendicular to the plane.