Exercise 0. Make sure you know how to do the problems from yesterday.

Exercise 1. A set $x$ is ordinal-definable if there are a formula $\phi$ in the language of set theory and finitely many ordinals $\alpha_0, \ldots, \alpha_n$ such that $x = \{y : \phi(y, \alpha_0, \ldots, \alpha_n)\}$.

(a) Reflect on why it isn’t obvious that the class of ordinal-definable sets is a definable class.

(b) Show that, nevertheless, a set is ordinal-definable iff there are a formula $\phi$ and ordinals $\beta$ and $\alpha_0, \ldots, \alpha_n < \beta$ such that $x = \{y \in V_\beta : V_\beta \models \phi(y, \alpha_0, \ldots, \alpha_n)\}$. (So it is a definable class, after all.)

Exercise 2. Show that if $\kappa$ is a strongly inaccessible cardinal in $M$ and $P$ is a poset of cardinality $< \kappa$ (in $M$), then $\kappa$ remains strongly inaccessible in any generic extension $M[G]$ by $P$.

Exercise 3. Suppose that $P$ and $Q$ are posets in $M$. Prove that a filter $F \subseteq P \times Q$ is $P \times Q$-generic over $M$ iff there are filters $G, H$ such that $F = G \times H$, $G$ is $P$-generic over $M$, and $H$ is $Q$-generic over $M[G]$. (In this case, $M[F] = M[G][H]$.)

Exercise 4. Suppose that $M$ is a transitive model of ZFC and that

$M \models \text{"}X \text{ and } Y \text{ are closed subsets of the unit interval } [0,1]\text{"}$.

(You may assume if you wish that $M \cap [0,1]$ is countable, though that shouldn’t be necessary.) Notice that the sets $M$ thinks are closed are unlikely to be closed. Prove that

(a) $\mu^M(X) = \mu(\overline{X})$.

(b) if $X \cap Y = \emptyset$, then $\overline{X} \cap \overline{Y} = \emptyset$.

(c) $\overline{X \cap Y} = \overline{X} \cap \overline{Y}$.

(Here $\overline{X}$ is the closure of $X$ in the true unit interval.)

Email Zach if you have questions or need a hint.

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