Exercise 1: Evaluate $r'(g(t))$ if $r(t) = (t^2, 1 - t, \sin(t))$ and $g(t) = e^t$.

$$r'(t) = (2t, -1, \cos(t)) \quad r'(g(t)) = (2e^t, -1, \cos(e^t))$$

Exercise 2: Does any tangent line to the curve parameterised by $r(t) = (t - t^3, 12 - t^2, 3 - t)$ intersect the $x$-axis?

Tangent vector: $p'(t) = (-3t^2, -2t, -1)$

Tangent line equation: $x(s) = p(t) + s v'(t)$

Exercise 3: Find the length of a helix with radius $R$ and pitch $2\pi P$ that makes $N$ complete turns. The parametric equation of such a helix is

$$r(t) = (R \cos(t), R \sin(t), Pt).$$

$$\int_0^{2\pi N} ||p'(t)|| dt = 2\pi N \sqrt{R^2 + P^2}$$

Exercise 4: Find a parameterisation of the line $y = mx$. Using this parameterisation, find an arc-length parameterisation of the line.

Parameterisation: $p(t) = (t, mt)$

Arc-length parameterisation: $p(s) = \left( \frac{s}{\sqrt{1+m^2}}, \frac{ms}{\sqrt{1+m^2}} \right)$