Math 33B  Differential Equation
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Office hour: 10:00-11:00 am, Tue NS 6118
Notice:
1. Worksheet is collected and graded after class every week (after this week).
2. Please don’t ask me about homework, because I don’t grade them.
3. Any questions and suggestions about this class is welcomed. Feel free to send me email or make an appointment.

Today’s discussion note:
- A brief review of 32A and 32B.
  32A  32B
  (Differentiation)  (Integration).
  • Vector valued functions  • Line integral
  • Curve geometry  • Surface integral
  • Multivariable functions  • Double/triple integral
  • Contour plot.
- Introduction to differential equation.
  \[
  \frac{dy}{dt} = f(t,y), \quad y(0)=2
  \]
  1. If \( f=f(t) \), \( y(t)=f(t) \Rightarrow y(t)=y(0)+\int_0^t f(t)\,dt \).
  Turns into an integration problem.
  Initial value is necessary for solution.
  2. If \( f(t,y)=y-1 \), separate variables.
  \[
  \frac{dy}{dt} = y-1 \Rightarrow \frac{dy}{y-1} = dt \Rightarrow \log\frac{y}{y-1} = t+
  \]
  \( y(t) = 1 + e^{t(y(0)-1)} \).
  Solution is exponential function.
3. \( f(t, y) = (y-1)^2 \)

\[
\frac{dy}{dt} = (y-1)^2 \Rightarrow \frac{dy}{(y-1)^2} = dt = -\frac{1}{y-1} dt \\
\therefore \quad y(t) = 1 + \frac{1}{y-1} t
\]

Say \( y(0) = 2 > 1 \), then \( y(t) = 1 + \frac{1}{1-1} t = 1 - \frac{2}{t-2} \). 

Solution explodes at \( t = 1 \).

That means the rate of decreasing goes to infinity, as \( t \to \infty \).

Now let's see some classification of differential equations

1. **Ordinary** --- \( \frac{dy}{dt} = f(t, y) \).
   
   **ODE** \( \frac{dy}{dt} = f(t, y, y', y'', \ldots, y^{(m)}) \).

2. **Partial** --- \( \frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} \) heat equation.

   **PDE** \( \frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} \) wave equation

   \( \frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = f \) Poisson equation.

This class will focus on ODE.

3. **Scalar equation** --- \( \frac{dy}{dt} = f(t, y) \)

4. **System of equation** --- \( \frac{dy_1}{dt} = f(t, y_1, y_2, \ldots, y_m) \)

5. **Normal form** of scalar equation.

   \( \frac{dy}{dt} = f(t, y, y', y'', \ldots, y^{(m)}) \)

6. **General form** of scalar equation.

7. \( n \) is called "order" of equation.

   e.g. \( \frac{dy}{dt} = 2y \frac{dy}{dt} + y^2 \) is second order.

   \( \frac{dy}{dt} = y - 1 \) is first order.
Now we come to first order equation.

Normal form: \( y' = f(t, y) \)

A geometric view of first order equation is direction field. \( f(t, y) \) is plotted as slope of \( y \).

\[ \frac{dy}{dt} = y \]

Given any initial value \( y(0) \), we can plot the solution curve by following the direction field.

\[ \frac{dy}{dt} = 1 - y^2 \]

First we notice there are two constant solutions:

\( y(t) = -1 \) and \( y(t) = 1 \).

From solution curves we can see,

\( y(t) = -1 \) is not "attractive", but \( y(t) = 1 \) is "attractive".