Relative meanings and other (unexpected) applications of the synonymy calculus

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Deduction in semantics workshop, 10 October, 2007

Referential uses of definite descriptions

Maureen Dowd in a recent opinion column in the New York Times, uses

> *President Bush, Mr. Bush, W., the President, the Texas President, he, him*

to refer to the same person

Claim: *In the context of the Dowd column, these phrases are synonymous*

- I would use them interchangeably in a report of the column
- I would use them interchangeably in a translation of the column to Greek
- In a translation of a Greek column to English, I would translate πλανητάρχης (literally *planet master*) as the US president or Busch

Donnellan 1966 on Linsky's example

Her husband is kind to her

...asserted on seeing a man (Smith) treating kindly a spinster, in the mistaken (perhaps shared) belief that he is her husband

Donnellan:

> *It seems to me that we shall, on the one hand, want to hold that the speaker said something true, but be reluctant to express this by "It is true that her husband is kind to her"*
>
> *This shows, I think, a difficulty in speaking simply about "the statement" when definite descriptions are used referentially*

Claim: *In this utterrance, her husband is synonymous with Smith and so the utterance is true*

Outline

(1) Formal Fregean semantics
(and there will be remarks on them throughout)

(2) Referential intension theory

(3) The significance and use of truth values

(4) Meaning and synonymy relative to linguistic conventions

1. *Sense and denotation as algorithm and Value*, 1994
2. *A logical calculus of meaning and synonymy*, 2006
3. (with E. Kalyvianaki) Two aspects of situated meaning, submitted

These are posted on my homepage
http://www.math.ucla.edu/∼ynm

Frege on sense

"*[the sense of a sign] may be the common property of many people*" Meanings are public (abstract?) objects

"*The sense of a proper name is grasped [wird erfasst] by everyone who is sufficiently familiar with the language . . . Comprehensive knowledge of the thing denoted . . . we never attain*"
Speakers of the language know the meanings of terms

"*The same sense has different expressions in different languages or even in the same language*"

"*The difference between a translation and the original text should properly not overstep the [level of the idea]*"
Faithful translation should preserve meaning

The methodology of formal Fregean semantics

- An *interpreted formal language* L is selected
- The rendering operation on a fragment of English:

  English expression + informal context

  $$\xrightarrow{\text{render}} \text{formal expression} + \text{state}$$

- Semantic values (denotations, meanings, etc.) are defined rigorously for the formal expressions of L and assigned to English expressions via the rendering operation
- Claim: L *should be a programming language*
  Slogan: English as a programming language

The typed λ-calculus with recursion $\mathsf{L}^\lambda_r(K)$

An extension of the typed λ-calculus, into which Montague's Language of Intensional Logic LIL can be easily interpreted (by Gallin)

Basic types $b \equiv e \mid t \mid s$ (entities, truth values, states)

Types: $\sigma :\equiv b \mid (\sigma_1 \to \sigma_2)$

Abbreviation: $\sigma_1 \times \sigma_2 \to \tau \equiv (\sigma_1 \to (\sigma_2 \to \tau))$

Every non-basic type is uniquely of the form

$$\sigma \equiv \sigma_1 \times \cdots \times \sigma_n \to b$$

$\mathrm{level}(b) = 0$
$\mathrm{level}(\sigma_1 \times \cdots \times \sigma_n \to b) = \max\{\mathrm{level}(\sigma_1), \ldots, \mathrm{level}(\sigma_n)\} + 1$

$\mathsf{L}^{\lambda}_{r}(K)$ - syntax

Pure Variables: v_0^σ, v_1^σ, ..., for each type σ $(v : \sigma)$
Recursive variables: p_0^σ, p_1^σ, ..., for each type σ $(p : \sigma)$
State parameters: $\bar{a}$ for each state a (for convenience only)

Constants: A finite set K of typed constants

Terms – with assumed type restrictions and assigned types $(A : \sigma)$

$$A :\equiv v \mid \bar{a} \mid p \mid c \mid B(C) \mid \lambda(v)(B)$$
$$\mid A_0 \text{ where } \{p_1 := A_1, \dots, p_n := A_n\}$$

$$C : \sigma, B : (\sigma \to \tau) \implies B(C) : \tau$$
$$v : \sigma, B : \tau \implies \lambda(v)(B) : (\sigma \to \tau)$$
$$A_0 : \sigma \implies A_0 \text{ where } \{p_1 := A_1, \dots, p_n := A_n\} : \sigma$$

Abbreviation: $A(B, C, D) \equiv A(B)(C)(D)$

$\mathsf{L}^\lambda_r(K)$ - denotational semantics

- We are given basic sets $\mathbb{T}_s, \mathbb{T}_e$ and $\mathbb{T}_t \subseteq \mathbb{T}_e$ for the basic types

$$\mathbb{T}_{\sigma\to\tau} = \text{the set of all functions } f : \mathbb{T}_\sigma \to \mathbb{T}_\tau$$
$$\mathbb{P}_b = \mathbb{T}_b \cup \{\bot\} = \text{the ``flat poset'' of } \mathbb{T}_b$$
$$\mathbb{P}_{\sigma\to\tau} = \text{the set of all functions } f : \mathbb{T}_\sigma \to \mathbb{P}_\tau$$

Each $\mathbb{P}_\sigma$ is a complete poset (with the pointwise ordering)

- We are given an object $c = \bar{\mathsf{c}} : \mathbb{P}_\sigma$ for each constant $\mathsf{c} : \sigma$
  - Pure variables of type σ vary over $\mathbb{T}_\sigma$; recursive ones over $\mathbb{P}_\sigma$
  - If $A : \sigma$ and π is a type-respecting assignment to the variables, then $\mathrm{den}(A)(\pi) \in \mathbb{P}_\sigma$
  - Recursive terms are interpreted by the taking of least-fixed-points

Rendering natural language in $L_r^\lambda(K)$

$$\text{Abelard loves Eloise} \xrightarrow{\text{render}} \text{loves(Abelard,Eloise)} : \tilde{t}$$

$$\text{Bush is the president} \xrightarrow{\text{render}} \text{eq(Bush,the(president))} : \tilde{t}$$

$$\text{liar} \xrightarrow{\text{render}} p \text{ where } \{p := \neg p\} : t$$

$$\text{truthteller} \xrightarrow{\text{render}} p \text{ where } \{p := p\} : t$$

$$\tilde{t} \equiv (s \to t) \qquad \text{(type of Carnap intensions)}$$

$$\tilde{e} \equiv (s \to e) \qquad \text{(type of individual concepts)}$$

$$\text{Abelard, Eloise, Bush} : \tilde{e}$$

$$\text{president} : \tilde{e} \to \tilde{t}, \text{eq} : \tilde{e} \times \tilde{e} \to \tilde{t}$$

$$\neg : \tilde{t} \to \tilde{t}, \text{the} : (\tilde{e} \to \tilde{t}) \to \tilde{e}$$

$$\text{den(liar)} = \text{den(truthteller)} = \perp$$

Can we say nonsense in $\mathsf{L}_r^\lambda(K)$?

Yes!

In particular, we have variables over states—so we can explicitly refer to the state (even to two states in one term); LIL does not allow this, *because we cannot do this in English*

Consider also the term

$$A \equiv \mathsf{rapidly}(\mathsf{tall})(\mathsf{John}) : \tilde{t}$$

(John is rapidly tall? John talls rapidly?)

— only A is already a LIL-term

- Distinct grammatical categories are mapped onto the same type (both in LIL and in $\mathsf{L}_r^\lambda(K)$), and so we can "say nonsense" in both formal languages

. . . and there is nothing wrong with this

Rendering natural language in $L_r^\lambda(K)$

John stumbled and fell $\xrightarrow{\text{render}} \lambda(x)\Big(\text{stumbled}(x)\ \&\ \text{fell}(x)\Big)(\text{John})$

(predication after coordination)

This is in Montague's LIL, the Language of Intensional Logic (as it is interpreted in $L_r^\lambda(K)$)

John stumbled and he fell $\xrightarrow{\text{render}} \text{stumbled}(j)\ \&\ \text{fell}(j)$ where $\{j := \text{John}\}$

(conjunction after co-indexing)

The logical form of this sentence cannot be captured faithfully in LIL — recursion models co-indexing preserving logical form

Meaning in $\mathsf{L}_r^\lambda(K)$

- In slogan form: *The meaning of a term A is faithfully modeled by an algorithm* int(A) *which computes den*(A)(π) *for every assignment* π
- The referential intension int(A) is (compositionally) determined from A
- int(A) is an abstract (not necessarily implementable) recursive algorithm which can be defined in $\mathsf{L}_r^\lambda(K)$
- Referential synonymy: $A \approx B \iff \text{int}(A) \sim \text{int}(A)$ (where $\sim$ is a natural isomorphism relation between abstract, recursive algorithms)
- Claim: *Meanings are faithfully modeled*
- Claim: *Synonymy is captured* (defined)

Is this notion of meaning Fregean?

Evans (in a discussion of Dummett's similar, computational interpretations of Frege's sense):

> *"This leads [Dummett] to think generally that the sense of an expression is (not a way of thinking about its [denotation], but) a method or procedure for determining its denotation. So someone who grasps the sense of a sentence will be possessed of some method for determining the sentence's truth value*
> *...ideal verificationism*
> *...there is scant evidence for attributing it to Frege"*

Converse question: If you posses a method for determining the truth value of a sentence A, do you then "grasp" the sense of A? (Sounds more like Davidson rather than Frege)

The Reduction Calculus, Canonical Forms

- A reduction relation $A \Rightarrow B$ is defined on terms of $\mathsf{L}^{\lambda}_{r}(K)$
- Each term A is effectively reducible to a unique (up to congruence) irreducible recursive term, its canonical form

$$A \Rightarrow \mathrm{cf}(A) \equiv A_0 \text{ where } \{p_1 := A_1, \ldots, p_n := A_n\}$$

- $\boxed{\mathrm{int}(A) = (\mathrm{den}(A_0), \mathrm{den}(A_1), \ldots, \mathrm{den}(A_n))}$
- The parts $A_0, \ldots, A_n$ of A are irreducible, explicit terms (the "truth conditions" of A)
- Claim: $\mathrm{cf}(A)$ *is the logical form of A*
- *Synonymy Theorem.* $A \approx B$ *if and only if*

$$B \Rightarrow \mathrm{cf}(B) \equiv B_0 \text{ where } \{p_1 := B_1, \ldots, p_m := B_m\}$$

so that $n = m$ *and for* $i \leq n$, $den(A_i) = den(B_i)$

Utterances and local meaning

- A *sentence* is a closed, parameter-free term $S : \tilde{t}$ (which denotes a *Carnap intension*, i.e., a function from states to truth values)
- An *utterance* is a pair (S, a) of a sentence $S : \tilde{t}$ and a state a; it is expressed in $\mathrm{L}^{\lambda}_{r}(K)$ by the term $S(\bar{a}) : t$
- The *local meaning* of a sentence S at a state a is $\mathrm{int}(S(\bar{a}))$, the referential intension of the utterance
  Local meanings are the objects of knowledge, belief, etc.
- Every term of pure type $S : t$ is synonymous with an utterance $S'(\bar{a})$ (so that mathematical claims can be known, believed, etc.)
- Kalyvianaki introduces the factual content of a sentence S at a state a, another semantic value which captures "what S says about the world at state a"

Presuppositions and errors

the King of France is bald $\xrightarrow{\text{render}}$ $BKF \equiv \text{bald}(\text{the}(\text{king of France}))$

- What is the truth value of $BKF(a)$ when a is today's state?
- Frege would leave it undefined
- Russell would make it false
- Executed as a program, $BKF(\bar{a})$ would return an error

using the definition

$$the(p)(a) = \begin{cases} \text{the unique } x \text{ such that } p(b \mapsto x)(a), \\ \qquad \text{if one such } x \text{ exists,} \\ er, \quad \text{otherwise} \end{cases}$$

where *er* is a "truth value" signifying "false presupposition"

Meaning under presupposition

$$BKF(\bar{a}) \Rightarrow \mathrm{bald}(k)(\bar{a}) \text{ where } \{k := \mathrm{the}(p),$$
$$p := \text{king of}(f), f := \text{France}\}$$

Execution of the algorithm $\mathrm{int}(BKF(\bar{a}))$ successively computes:

(1) $f := \mathrm{den}(\text{France})$, so that for every state b, $f(b) = \mathit{France}$

(2) $p := \mathrm{den}(\text{king of France})$, so that for every state b,
$p(b) \iff p(b)$ *is king of France*

(3) $k := \mathrm{den}(\mathrm{the}(\text{king of France}))$, so that for every state b,
$k(b) = \mathit{the}(\mathit{king\ of\ France})(b)$

(4) $\mathrm{den}(BKF(\bar{a})) := \mathrm{den}(\mathrm{bald}(k))(a) = \mathit{bald}(k)(a) = \mathit{er}$

Claim: *Knowing this algorithm is tantamount to understanding the utterance $BKF(\bar{a})$*,

...and the "truth value" which is returned is of some (but little) significance

Truth values galore

- den(the king of France is bald, a)
  $= er$ (there is no king of France)
- den(his wife is beautiful, a) $= er$ (he has two wives)
- den(is snow white?, a) $=$?true
- den(is the king of France bald?, a) $=?er$
- ...

Perhaps also

$$a \text{ but } b = \begin{cases} \text{true}, & \text{if } a = b = \text{true}, \\ \text{false}, & \text{if } a = \text{true}, b = \text{false}, \\ er, & \text{otherwise} \end{cases}$$

by which $\boxed{A \text{ but } B \not\approx A \text{ and } B}$, for any A, B

Meaning and entailment

Consider the following statement at the current state:

If Hamlet is bald, then snow is black

Is it true or false?

- The entailment is problematic
- The meaning (as algorithm) is clear

Claim: *Entailment is a poor guide to meaning*

—and in many cases it is irrelevant

…(The logic of natural language is certainly many-valued and most likely quite complex)

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Relativized meaning and synonymy

- A *lingustic convention* is a tuple $\mathcal{C} = (C_1, \ldots, C_k, w)$, where
  - $C_1, \ldots, C_k$ are closed terms of the same type σ
  - $w \in \mathbb{P}_\sigma$
- The *relativization* of a term A to $\mathcal{C}$ is the term

$$A_{\mathcal{C}} \equiv A\{C_1 :\equiv c\} \cdots \{C_k :\equiv c\} \quad (c \text{ a fresh constant}, c : \sigma)$$

- The *denotation of A* relative to $\mathcal{C}$ is $\mathrm{den}(A_{\mathcal{C}})$ in the expanded language with $\bar{c} = w$
- The *referential intension* of A relative to $\mathcal{C}$ is $\mathrm{int}(A_{\mathcal{C}})$
- *Synonymy* relative to a convention:

$$\begin{aligned} A \approx_{\mathcal{C}} B \iff & A_{\mathcal{C}} \approx B_{\mathcal{C}} \\ \iff & \mathrm{cf}(A_{\mathcal{C}}) \equiv A_0 \text{ where } \{p_1 := A_1, \ldots, p_n := A_n\} \\ & \mathrm{cf}(B_{\mathcal{C}}) \equiv B_0 \text{ where } \{p_1 := B_1, \ldots, p_n := B_n\} \\ & \text{and } \mathrm{den}(A_0) = \mathrm{den}(B_0), \ldots, \mathrm{den}(A_n) = \mathrm{den}(B_n) \end{aligned}$$

Synonymy relative to a convention $(C_1, \ldots, C_n, w)$

- If $\mathcal{C} = ($Bush, the President, he, W., $\lambda(a)$Bush$)$, then

  Bush is ignorant $\approx_{\mathcal{C}}$ the President is ignorant $\approx_{\mathcal{C}} \cdots$

- If $\mathcal{C} = ($Hamlet, the Prince of Denmark, *er*$)$, then

  Hamlet was depressed $\approx_{\mathcal{C}}$ the Prince of Denmark was depressed

In Greek:
μπατζανάκηδες$(x, y) \iff x$ and y are married to sisters

- If $\mathcal{C} = ($μπατζανάκηδες, brothers in law, brothers in law$)$,then

  Ο Νιάρχος και ο Ωνάσσης ήταν μπατζανάκηδες

  $\approx_{\mathcal{C}}$ Niarchos and Onassis were brothers in law

Relative meaning and denotation

- If $\mathrm{den}(C_1) = \cdots = \mathrm{den}(C_n) = w$, then $\mathrm{den}(A) = \mathrm{den}(A_{\mathcal{C}})$
- If $\mathrm{den}(C_1)(a) = \cdots = \mathrm{den}(C_n)(a) = w$ and $C_1, \ldots, C_n$ *occur locally* in A, then $\mathrm{den}(A)(a) = \mathrm{den}(A_{\mathcal{C}})(a)$
- In general, $\mathrm{den}(A)(a) \neq \mathrm{den}(A_{\mathcal{C}})(a)$, and it may be that

$$A(a) \approx_{\mathcal{C}} B \text{ but } \mathrm{den}(A)(a) \neq \mathrm{den}(B)(a)$$

Amendment to the basic setup

The basic rendering operation becomes

$$\begin{aligned}&\text{English expression} + \text{informal context}\\&\qquad\xrightarrow{\text{render}} \text{formal expression} + \text{linguistic conventions} + \text{state}\end{aligned}$$

which determine global and local meaning relative to the conventions

- The relativization operation relative to a set of linguistic conventions is very similar to coordination, but it uses a fresh constant rather than a variable

The (mythical) language speaker of Frege

must know the linguistic part of the rendering operation,

$$\begin{aligned}&\text{English expression} + \text{informal context}\\ &\qquad\xrightarrow{\text{render}} \text{formal expression} + \text{linguistic conventions}\end{aligned}$$

and the reduction calculus, which constructs the algorithm that computes the value of a given term at a given state

To compute the value of a term, i.e., to gain "*comprehensive knowledge of the thing denoted*", requires knowledge of the values of the constants and (in many cases) infinite computing power;

. . . and this "*we never attain*"