Additional problems

This is a dynamic list of problems, with a few problems periodically added as the class progresses; they will be referred to in the Homework assignments by their numbers here, a1, a2, etc.

For Problems a1 and a2 the references are to Part 1 of the Notes, the *Propositional Calculus* PL.

a1. With the notation for assignments on page 6 and the notation for replacement on page 8, set for each assignment v and each list of distinct variables $p_1, \dots, p_k$ and formulas $\psi_1, \dots, \psi_k$,

$$v^*(\mathsf{A}_j) = \begin{cases} \overline{v}(\psi_i), & \text{if } \mathsf{A}_j \equiv p_i, \text{ for some } i, \\ v(\mathsf{A}_j), & \text{otherwise,} \end{cases}$$

and prove that for every formula ϕ,

$$(\star) \qquad \overline{v}(\phi\{p_1 :\equiv \psi_1, \dots, p_k :\equiv \psi_k\}) = \overline{v^*}(\phi).$$

Hint: Use structural induction and the Tarski conditions in Theorem 3F.1 which determine the function $\overline{v}$, so that you do not need to appeal to the "official" definition of $\overline{v}$ on page 6.

a2. Use Problem a1 to prove the Replacement Theorem 2D.1.

From this point on, the references are to Part 2 of the Notes, the *Lower Predicate Calculus with Identity* LPCI

a3. Outline a proof of Proposition 2B.1, the Parsing Lemma for LPCI-terms. Hint: Prove first that *no term can be a proper initial part of another term*, and then use induction on the length of terms.

a4. Outline a proof of Proposition 2B.2, the Parsing Lemma for LPCI-formulas. Hint: Show first the number of left parentheses matches the number of right parentheses in a formula; that if ϕ is a formula and $\alpha \sqsubseteq \phi$, then the number of left parentheses in α is greater than or equal to the number of right parentheses in α; and that no formula is a proper, initial segment of another. The result follows from these Lemmas by induction on the length of formulas.