Math 3a Week 3 Discussion Practice Problems January 25, 2017

Please show all your work! Answers without supporting work will not be given credit.

Name:______________________________

1. Warm up

• The number of Instagram followers I had on January 1 2016 was 2 and the number I had on January 1 2017 was 150. If the population grows linearly, when will I have 1000 followers? What about if it grows exponentially?

• Let \( f(x) = \ln(x) \) and \( g(x) = e^{3x+1} \). Simplify the compositions \( f(g(x)) \) and \( g(f(x)) \)?

• Find the inverse function of \( f(x) = 2^x \)? What are the domain and range of \( f \) and its inverse?

2. (Exponential Decay, Half-life) Carbon-14 has a half-life of 5730 years. What is the decay constant? You are presented with a document which purports to contain the recollections of a Mycenaean soldier during the Trojan War. The city of Troy was finally destroyed in about 1250 BC, or about 3250 years ago. Carbon-dating evaluates the ratio of radioactive carbon-14 to stable carbon-12. Given the amount of carbon-12 contained a measured sample cut from the document, there would have been about \( 1.3 \times 10^{-12} \) grams of carbon-14 in the sample when the parchment was new, assuming the proposed age is correct. According to your equipment, there remains \( 1.0 \times 10^{-12} \) grams. Is there a possibility that this is a genuine document? Or is this instead a recent forgery? Justify your conclusions.
3. Earthquake intensity is measured by the Richter scale. The formula for the Richter rating of a given quake is given by

$$R = \log\left(\frac{I}{I_0}\right)$$

where $I_0$ is the "threshold quake", or movement that can barely be detected, and the intensity $I$ is given in terms of multiples of that threshold intensity.

You have a seismograph set up at home, and see that there was an event while you were out that had an intensity of $I = 989I_0$. What is the intensity of this quake on the Richter scale? How much stronger in terms of earth movement is an 8.0 quake compared to a 3.0 quake?

The moment magnitude scale was developed in the 1970s to succeed the Richter magnitude scale. Even though the formulas are different, the new scale retains a similar continuum of magnitude values to that defined by the older one. As with the Richter magnitude scale, an increase of one step on this logarithmic scale corresponds to a $10^{1.5}$ (about 32) times increase in the amount of energy released, and an increase of two steps corresponds to a $10^3$ times increase in energy. Write a formula for this new scale.  

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1 Problems 2 and 3 are adapted from purplemath.com