Please show **all** your work! Answers without supporting work will not be given credit.

Name: _______________________

1. Spacetime Warner Cable charges for Internet service according to the formula: \( C(n) = 40 + \frac{1}{50}n \) where \( n \) is kilobits of data and \( C(n) \) is the monthly charge in dollars. Find and interpret the rate of change and initial value.

The rate of change is \( \frac{1}{50} \) which is the cost in dollars per kilobit used. The initial value is \( 40 \) which is the base charge of having Spacetime Warner Cable regardless of usage.

2. Using the knowledge that the conversion between Celsius and Fahrenheit is linear and that \( 32^\circ F = 0^\circ C \) and \( 212^\circ F = 100^\circ C \), find an equation that computes Celsius temperature in terms of Fahrenheit. Find the inverse equation that computes Fahrenheit in terms of Celsius. Is there a temperature that is the same in both Fahrenheit and Celsius?

Because the relationship is linear and we have two points we are able to compute the conversion formula. The two points we have are \((32, 0)\) and \((212, 100)\) where we are thinking of putting Fahrenheit on the horizontal axis and Celsius on the vertical axis. With this we get \( C - 0 = \frac{100 - 0}{212 - 32}(F - 32) \), i.e. \( C = \frac{5}{9}(F - 32) \) as an equation that takes Fahrenheit temperatures as input \( F \) and outputs Celsius temperatures \( C \). To find the equation that takes \( C \) as input and \( F \) as output, we can repeat the procedure using the points \((0, 32)\) and \((100, 212)\) or we can isolate \( F \) in the equation we already found \( C = \frac{5}{9}(F - 32) \). Either way, we get \( F = \frac{9}{5}C + 32 \). To find a temperature that is both the same in Fahrenheit and Celsius, we force \( C \) to be equal to \( F \) in either equation we found, e.g. \( F = \frac{5}{9}(F - 32) \) and solve for \( F \), which gives \( 40^\circ \). We can do this by forcing \( F \) to be equal to \( C \) too.

3. What are the horizontal and vertical intercepts of \( 5x + 5y = 10 \)

To find horizontal intercepts, also known as \( x \) intercepts in this context, we set \( y = 0 \) and solve for \( x \). We find \( x = 2 \) and so our final answer is that there is a horizontal intercept at the point \((2, 0)\). Similarly to find the vertical intercept (I say the vertical intercept because functions can have at most one due to the vertical line test), we set \( x = 0 \) and solve for \( y \). We get that there is vertical intercept at the point \((0, 2)\). Note it is wrong to say the intercept is just 2, as intercepts are points and thus we need to give the full coordinates of the points.
4. Ride-sharing companies Doober and Dryft offer two different pricing schemes. With Doober, there is a fixed charge of $3 plus $0.40 per mile. With Dryft, there is a fixed charge of $1 plus $0.50 per mile. Which company should you use for shorter rides and which company should you use for longer rides? What’s the break-even point when it doesn’t matter which company you use?

Let \( P(x) = 1 + 0.5x \) be the equation that takes as input miles \( x \) and outputs the Dryft price and \( Q(x) = 3 + 0.4x \) be the equation that inputs miles \( x \) and outputs Doober price. To find when Dryft costs less money than Doober, we solve the inequality \( P(x) < Q(x) \), i.e. \( 1 + 0.5x < 3 + 0.4x \). We obtain that \( x < 20 \), which we interpret as the price of Dryft is less than the price of Doober when our journey is less than 20 miles (of course to be sensible, our journey must not be negative). To find when Doober costs less than Dryft we solve \( Q(x) < P(x) \) and obtain \( x > 20 \). Thus the price of Doober is less than the price of Dryft when the journey is more than 20 miles. To find when the cost will be the same, we set \( P(x) = Q(x) \) and we get \( x = 20 \), i.e. when the journey is 20 miles, both will cost \$11 as \( P(20) = Q(20) = 11 \). We can also consider the problem graphically by plotting both functions. We see that initially the Dryft price function is lower when \( 0 < x < 20 \) and then the functions intersect at \( x = 20 \) and then the Doober price function is now lower.

5. The number of followers I have on popular social media platform, MyFace, was 4000 in 2010, and then 6500 in 2012. Assuming linear growth, how many followers will I have in 2016? When will I have 10000 followers? When did I sign up for MyFace?

In this problem, we have a choice to make, we can let our independent time variable \( x \) be measured in the number of years since 2010 or we let \( x \) measure the actual year it is. Because it’s not sensible to talk about the number of MyFace followers I have a millenium before I’m born, I recommend we let \( x \) represent years since 2010. Now we repeat the procedures from above by using the points we are given to write down the function that takes \( x \) as input and outputs number of followers. The points we have are (0, 4000) and (2, 6500). Thus we obtain \( f(x) = 1250x + 4000 \). To find out how many followers I have in 2016, 6 years after 2010, we compute \( f(6) = 11500 \). To find out when I will have 10000 followers, we set \( f(x) = 10000 \) and obtain \( x = 4.8 \) so it was sometime during late 2014. To see when I signed up for MyFace, assuming you start with no followers, we set \( f(x) = 0 \) and obtain \( x = -3.2 \), and thus it was during late 2006 (2010-3.2=2006.8) that I signed up.

6. Plot my number of followers from the above problem with the year on the horizontal axis.
7. Solve $|x + 5| = 2$ for $x$

The equation $|x + 5| = 2$ leads us to two equations, one where we consider the inside of the absolute value bars already positive and thus we just remove them $x + 5 = 2$ which gives $x = -3$. The other is where we consider the inside of the absolute value bars to be negative and thus the absolute value negates the negative number so we have $-(x + 5) = 2$ which gives $x = -7$. So our final answer is $x = -7$ or $x = -3$.

8. Solve $2|4 - x| = -\frac{1}{2}$ for $x$

Dividing both sides by 2 gives $|4 - x| = -\frac{1}{4}$. As the absolute value of a number can never be negative, we have that regardless of what we plug in for $x$, the quantity $|4 - x|$ will always be nonnegative. Thus there are no solutions.

9. Solve $|6x| = |-6|$ for $x$

As $-6$ is just a constant we can simplify the right hand side to 6 and thus we have to solve $|6x| = 6$. So our final answer is $x = -1$ or $x = 1$.

10. Solve $\frac{1}{2}|2x + 2| - 15 = 1$ for $x$

It is usually a good idea to isolate the absolute value. Rearranging the equation to get $|2x + 2| = 32$ now leads us to two equations, one where consider the inside of the absolute value bars already positive and thus we just remove them $2x + 2 = 32$ which gives $x = 15$. The other is where the inside of the absolute value bars is negative and thus the absolute value negates the negative number so we have $-(2x + 2) = 32$ which gives $x = -17$. So our final answer is $x = -17$ or $x = 15$. Cont.
11. Solve $|x + 4| > 2$ for $x$

$|x + 4| > 2$ is really two separate inequalities, $x + 4 > 2$ or $x + 4 < -2$. They are separate in the sense that if on one hand $x + 4$ is greater than 2 then the inequality is satisfied. On the other hand if $x + 4$ is less than $-2$, then when taking the absolute value it becomes a positive number that is greater than 2 thus satisfying the inequality. Solving $x + 4 > 2$ gives $x > -2$ and solving $x + 4 < -2$ gives $x < -6$. So our final solution is $(-\infty, -6) \cup (-2, \infty)$.

12. Solve $4|3 - x| < -\frac{1}{5}$ for $x$

Isolating the absolute value gives $|3 - x| < -\frac{1}{20}$. As absolute values can never be negative, we again have no solutions.

13. Solve $|5x| \geq |7|$ for $x$

As $-7$ is just a constant we replace $|-7|$ with 7. Thus we have $|5x| \geq 7$ which is really two separate inequalities, $5x \geq 7$ or $5x \leq -7$. They are separate in the sense that if on one hand $5x$ is greater than 7 then the inequality is satisfied. On the other hand if $5x$ is less than -7, then when taking the absolute value it becomes a positive number that is greater than 7 thus satisfying the inequality. Solving $5x \geq 7$ gives $x \geq \frac{7}{5}$ and solving $5x \leq -7$ gives $x \leq -\frac{7}{5}$. So our final solution is $(-\infty, -\frac{7}{5}] \cup [\frac{7}{5}, \infty)$.

14. Solve $\frac{1}{2}|x + 2| - 1 \leq 10$ for $x$

Isolating the absolute value gives $|x + 2| \leq 22$, which is really two simultaneous inequalities, $-x + 2 \leq 22$ and $-x + 2 \geq -22$. They are simultaneous in the sense that we need $-x + 2$ to be both less than or equal to 22 as well as greater than or equal to -22, so that when taking the absolute value it becomes a positive number that is still less than 22 thus satisfying the inequality. Solving $-x + 2 \leq 22$ gives $x \geq -20$ and solving $-x + 2 \geq -22$ gives $x \leq 24$. So our final solution is $[-20, 24]$.

The End.