Homework Assignment 2 – Math 273a, Fall 2015

- This homework assignment is open to all textbooks (listed above or not), class notes, and Internet documents except for any solution manual or the solutions from the previous quarters.
- You are encouraged to ask questions and discuss the questions and solutions on http://piazza.com/ucla/fall2015/math273a/home. However, copying others solutions or programs is considered a **serious violation**.
- Please type your answers in Latex and **submit a single PDF file**. Note that you can insert an external PDF file to your latex file by using

```
\usepackage{pdfpages} % add this line before \begin{document}

\includepdf[pages={1,2,3}]{myfile.pdf}
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- Although not required, you are encouraged to use notation similar to the course slides.
- Please **do not include your name anywhere in the submitted PDF file**.

Problem 1. (10 points)

[Type the problem here, or give its reference.]

Answer: [Type your answer here. Make sure you clearly define all mathematical objects in the answer.]

Problem 2. (10 points)

[Start a problem on a new page. Type the problem here, or give its reference.]

Answer: [Type your answer here. Make sure you clearly define all mathematical objects in the answer.]

Problem 3. (10 points)

[Start a problem on a new page. Type the problem here, or give its reference.]

Answer: [Type your answer here. Make sure you clearly define all mathematical objects in the answer.]

Problem 4. (10 points)

Suppose that the polyderon $P = \{x \in \mathbb{R}^n : a_i^T x \geq b_i,\ i = 1, \ldots, m\}$ is nonempty. Show that the following statements are equivalent:

(a) P has a least one extreme point;

(b) P does not contain a line;

(c) There exist n vectors out of $a_1, \ldots, a_m$ that are linearly independent.

(Comments: P is not a standard–form polyhedron. A vector $x \in P$ is an *extreme point* if there do not exist two vectors $y, z \in P$, both different from x, and a scalar $\alpha \in (0, 1)$ such that $x = \alpha y + (1 - \alpha)z$. Given vectors x and d, $\{x + \lambda d : \lambda \in \mathbb{R}\}$ is a line. Each direction has 3 points. If all the three directions are correct, add one extra point.)

Answer: [Type your answer here. Make sure you clearly define all mathematical objects in the answer.]

Problem 5. (10 points)

Consider a linear program in the standard form. Let x be a basic feasible solution associated with the basis matrix B. Prove the following statements:

(a) If the reduce cost of every nonbasic variable is positive, then x is the *unique* solution;

(b) If x is the unique solution and is nondegenerate, then the reduced cost of every nonbasic solution is positive.

(5 points for each statement.)

Answer: [Type your answer here. Make sure you clearly define all mathematical objects in the answer.]