Variational Approaches to Image Fusion

Overview
- Introduction to image fusion
- Adaptive IHS pan-sharpening
- Variational wavelet pan-sharpening
- Variational density estimation

The Flood of Data
- Government agencies have to gather geospatial intelligence in many situations: military, environmental, mapping, agricultural, emergency response.
- Data comes in different modalities.
  - Images: Color (Google maps), Infrared, Hyperspectral
  - Image-like data: RADAR, LIDAR, gravimetric
  - Point data: census data, events (e.g. crimes)
  - Text: field reports, news reports, rumors
- Each modality offers different advantages.

Data Fusion
- Data fusion / data integration combines multiple datasets (or the analyses of the datasets) into one dataset (or analysis).
- The goal is to represent the major features of each dataset in the fused result.

Image Modalities
- A standard color image consists of 3 bands: RGB.
- A multispectral image is typically a 4-6 band image: RGB + one or more infrared bands.

Image Modalities
- A hyperspectral image typically has ~200 bands, each band representing the response to a precise wavelength of light.
- So each pixel is a 200-dimensional signal.
- The signal can potentially identify the material present.
Difficulty with spectral images
- It is difficult to build a camera with both high spectral and spatial resolution.
- As the camera sensors are fine-tuned to a specific wavelength of light, the sensor loses spatial accuracy.
- So obtaining the extra image bands comes at the price of "bigger pixels".

Pan-sharpening
- To compensate for the spectral / spatial trade-off, many earth-observing satellites are equipped with 2 types of sensors:
  - **Multispectral** -- a 4-band image with good spectral (color) information but low spatial detail
  - **Panchromatic** -- a grayscale (1-band) image with high spatial detail, but no spectral information.
- For example, in the Quickbird satellite the panchromatic image has 0.6m resolution and the multispectral image has 2.4m resolution.

IHS Pan-sharpening
- The standard pan-sharpening technique is the IHS (Intensity-Hue-Saturation) transform.
- For a multispectral image $M$ and a panchromatic image $P$, compute
  $$ F_i = M_i + P - I $$
  $$ I = \frac{1}{4} M_1 + \frac{1}{4} M_3 + \frac{1}{4} M_2 + \frac{1}{4} M_4 $$
IHS Pan-sharpening

- We can generalize the IHS model to arbitrary coefficients.

\[ F_i = P + M - 1 \]

\[ I = \alpha_1 M_1 + \alpha_2 M_2 + \alpha_3 M_3 + \alpha_4 M_4 \]

- Ideally, these coefficients would be derived from information about the sensor.

(Choi-Cho-Kim 2008) suggested experimentally determined values for the IKONOS satellite

\[ I = 0.1 M_1 + 0.25 M_2 + 0.0833 M_3 + 0.567 M_4 \]

Adaptive IHS Pan-sharpening

- Without knowing the sensor details, can we reverse engineer the coefficients from the image?

- We want to approximate the panchromatic image as a linear combination of the multispectral bands:

\[ I = \alpha_1 M_1 + \alpha_2 M_2 + \alpha_3 M_3 + \alpha_4 M_4 = \text{Pan} \]

- We calculate the coefficients which minimize the following function:

\[ E(\alpha) = \sum (\alpha_i M_i(x) - P(x))^2 + \frac{\lambda}{\|
abla e\|} \]

- Furthermore, we note that the image colors should match away from edges:

\[ F_i = M_i + e(x)(P - 1), \quad e(x) = \exp \left( \frac{\lambda}{\|
abla e\|} \right) \]

Adaptive IHS Pan-sharpening

- The adaptive IHS method gives the same spatial quality, but the spectral information (colors) match the original image better.

REU

- The REU student team wrote Matlab software which runs several pan-sharpening methods and evaluates the performance under a suite of quality metrics.

- Adaptive IHS outperforms the standard IHS fusion in terms of preserving the spectral information (Rahmani-Merkurjev-Straits-Moeller-W, 2009).

Variational Wavelet Pan-sharpening

- All things in nature seek out a lower energy state.

- The variational approach (also known as energy or PDE-based methods) prescribes an energy to an image.

- High energy = "Bad" image

- Low energy = "Good" image

- The hard part is picking a good energy.

- As an example, we might minimize the H1 norm.

\[ \min J[u] = \int_{\Omega} \| \nabla u \|^2 \, dx \]
A Variational Example
- For example, we might choose to minimize the $H_1$ norm of the image.
\[
\min_X J(X) = \int |\nabla X|^2 \, dx
\]
- This wipes out oscillations in the image.
- But it also wipes out the edges!
- A better choice is the Rudin-Osher-Fatemi Total Variation (TV) norm.
\[
\min_X J(X) = \int |\nabla X| \, dx
\]

Variational Pan-sharpening
- We can't trust the colors of the panchromatic image $P$, but we can use the edges.
- We need a term that connects our new image $X_n$ to the high-res panchromatic image $P$.
\[
\min \left\{ \| \nabla P \cdot \nabla X_n \| \right\}
\]
- The new image $X_n$ should show large change in the $n$ direction, but not in the direction of $t$ (Ballester-Caselles-Verdera-Rouge, 2006).

Edge Alignment
- So if we define the panchromatic image's unit normal to be:
\[
\theta = \frac{\nabla P}{|\nabla P|}
\]
- Then we want our new image $X_n$ to not change across this direction.
\[
\min \int_\Omega |P^t \cdot \nabla X_n| \, dx
\]
- Integrating by parts gives
\[
\min \int_\Omega \| \nabla X_n \| + |X_n \cdot \text{div} (\theta)| \, dx
\]

Wavelet Decomposition
- The wavelet decomposition separates the color and edge information.

Wavelet Fusion
- A popular pan-sharpening technique is to combine the wavelet coefficients to form a new image.
- But this mix-and-match approach generally produces strange image artifacts.

Wavelet Matching
- We propose matching the high level wavelet coefficients of the images.
\[
J(X_n) = \sum_{n=1}^N \gamma_n \int (|\nabla X_n| + \text{div}(\theta) \cdot X_n) \, dx + \sum_{(j,k)} \lambda_{(j,k)} (\beta_{(j,k)}^n - \gamma_{(j,k)})^2
\]
- This produces images with higher spatial quality.
- Method can extend to any number of bands.
VWP: Variational Wavelet Pan-sharpening

- Add terms to force $X_i$ to match the colors of the original image $X_i^S$.
- Given a multispectral image $X_i^S$, we produce a sharpened multispectral image $X_i$ by minimizing the energy:

$$J(X_i) = \sum_{i} \int_{\Omega} \left( \|\nabla X_i + \text{div} \mathbf{v} \| \cdot X_i \right) dx + \sum_{i} \lambda_i \left( \gamma_i(X_i) - \gamma_i(X_i^S) \right)^2 + \sum_{c=1}^{C} \lambda_c \int_{\Omega} \left( X_i^c + X_i^S - X_i^c + X_i^S \right) dx + \sum_{i} \int_{\Omega} \left( X_i - X_i^S \right)^2 dx$$

- Force the image $X_i$ to respect the edges of the panchromatic whose orientation is given by $\mathbf{v}$.
- The high level wavelet coefficients should match those of the panchromatic.
- Enforce spectral quality by matching the colors of $X_i$ to those in the original data $X_i^S$.

Extension to Hyperspectral Images

- VWP extends nicely to high dimensional hyperspectral images (Moeller-W-Bertozzi, 2009).

1. The model can handle any number of bands.
2. The master image does not need to be a panchromatic image. Any high resolution grayscale image will suffice. (We used pictures from Google Maps.)
3. The model explicitly enforces spectral quality, so the hyperspectral signals will be preserved as much as possible.

Hyper-sharpening

- We examined portions of the 82-band San Diego image.
- Since we didn't have a panchromatic image, we used a high resolution image from Google Maps.
- Images were aligned manually.
Hyper-sharpening
- Features are much more visible in the sharpened image.
- We believe this could potentially improve classification and detection algorithms.

Spectral Fidelity
- Away from edges, VWP preserves the original signal.
- On edges, the contrast is changed but the overall signal shape stays the same.

Image Registration
- If the images are not aligned precisely, the result will be blurred.
- But useful information can still be seen.
- We may be able to deblur and/or improve image registration using spectral unmixing (Guo-W-Osher, 2009).

Variational Density Estimation
Joint work with Laura Smith, Matt Keegan, George Mohler, Andrea Bertozzi

Density Estimation
- The goal of density estimation is to construct the underlying probability density from discrete event data (e.g., crimes, census data, temperatures).
- For event data attached to geographic features, we often visualize density estimates in thematic maps.
- We see these type of images on the news every day.
- Density estimation is more than just visualization.

Kernel Density Estimation
- The standard approach is kernel density estimation.
- This is essentially a Gaussian blur of the data.
- It does not take into account geographical features.
Kernel Density Estimation

- The standard kernel density estimate can be written as a Maximum Penalized Likelihood Estimation (MPLE) problem.

\[ \hat{u}(x) = \arg\min_{\hat{u}} \{ \int \nabla^2 \hat{u} \, dx - \mu \sum_{i} \log(u(x_i)) \} \]

where \( u(x) \) is our probability distribution and \( x_i \) are the event locations.
- Minimizing the H1 norm gives preferences to blurry regions between areas of different densities, instead of edges.

TV MPLE Model

- We propose 2 changes (Smith-Keegan-W-Mohler-Bertozzi, 2010).
  1. The TV norm \( \int |\nabla u| \, dx \) is more appropriate for showing changes in density.
  2. If we can detect "invalid" region \( D \) from geographic data, we can align the density \( u \) with the unit normal to \( D \)

\[ \hat{u}(x) = \arg\min_{\hat{u}} \{ \int |\nabla u| \, dx + \lambda \int_0^1 \theta \, d\theta \sum_{i} \log(u(x_i)) \} \]
- Note this term also appears in our VWP model.

Crime Data

- We obtained crime event data from the LAPD.

Crime data

- We can look up this scene on Google Earth.
  - San Fernando Valley Residential Burglary
  - Residential Burglaries

Crime data

- Preparing the validity mask from the aerial image is extremely difficult.
- We tried clustering the image data based on watershed transforms.
- We tried identifying regions based on textural cues derived from training data.
Crime data

- Fortunately, we found out where the houses were from the LA County Tax Assessor's Office.

That's All Folks!

- Thank you for listening.
- Please send questions or comments to wittman@math.ucla.edu
- Reports available at www.math.ucla.edu/~wittman/lambda (password on request)