• History of Mathematics  106

• Illustrations and Supplements
  Chapter 1
Some Early Mathematics

Middle East around 1900 BCE
Rhind Papyrus
1850 BCE

Calculations with Fractions
Linear Equations
Areas Volumes

Egypt
Base 10

\[ 142 = 1 \times 10^2 + 4 \times 10 + 2 \]

Base 60

\[ 142 = 2 \times 60 + 22 \]
Babylonian Numbers

**Sexagesimal System**

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No Zero!
Example

\[ 142 = 2 \times 60 + 22 \]
Question: How do you write 120?

Is this 120 or 2? Need a space marker or a zero.
Why base 60?

360 + 5 = \#days in a year

360 days = 12 months \times

30 days

Both 12 and 30 divide 60

60 = 12 \times 5 = 30 \times 2
Plimpton 322

1800 BCE
## Pythagorean Triples

\[
a^2 + b^2 = c^2
\]

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<th>a</th>
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<td>161</td>
<td>240</td>
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a=3   b=4   c=5
Pythagoras

About 580-500 BCE
Born in Samos

Had school in Kroton

Born in Samos
Pythagoreans:

• Used Deductive reasoning

• Connected abstract mathematics with Nature

"They supposed the elements of number to be the elements of all things, and the whole heavens to be a musical scale and a number."  Aristotle
Some Achievements

• Explained Musical Harmony in terms of fractions

• Pythagorean Theorem

• Discovery of irrational lengths
Legendary Beliefs

- Transmigration of Souls. Eat no beans since they came from same source as humans

- Pythagoras had a golden thigh (a sign of divinity); the people of Croton called him the Hyperborean Apollo (Apollo's avatar)
Pythagorean Theorem

\[ a^2 + b^2 = c^2 \]
Proof of Pythagorean Theorem

\[ c^2 = a^2 + b^2 \]
Another Proof of Pythagorean Theorem

\[(a+b)^2 = c^2 + 4(ab/2)\]
Outline of the rest of the chapter

• Review of basic properties of integers
• Formulas for Pythagorean Triples
• Irrationality of Square root of 2
• Similar Triangles
• Law of Cosines (Supplement)
Law of Cosines

\[ c^2 = a^2 + b^2 - 2ab \cos \theta \]

Proof:

\[ d = b \cos(\pi - \theta) \quad e = b \sin(\pi - \theta) \]

\[ c^2 = (a + d)^2 + e^2 = a^2 + b^2 (\sin^2(\pi - \theta) + \cos^2(\pi - \theta)) + 2ab \cos(\pi - \theta) \]

\[ = a^2 + b^2 - 2ab \cos \theta \]

For the proof we assume that the angle is obtuse