We want to prove the following inequality:

$$\frac{1}{2}(1 - cos1) \leq \iint_{[0,1]\times[0,1]} \frac{sinx}{1+(xy)^4} dA \leq 1$$

The inequality on the right follows directly from the Mean Value Theorem for Integrals, because $\frac{sinx}{1+(xy)^4} \leq 1$ for all x and y, and the area of $[0,1]\times[0,1]$ is 1. For the inequality on the left, note that on this domain, we have $(xy)^4 \leq 1$, which gives us the following:

$$\begin{aligned} 2 &\geq 1+(xy)^4 \\ \frac{1}{2} &\leq \frac{1}{1+(xy)^4} \\ \frac{1}{2}sinx &\leq \frac{sinx}{1+(xy)^4} \end{aligned}$$

Integrating the last line gives the desired result. (I'll leave it up to you to show that $\int_0^1 \int_0^1 \frac{1}{2} sinxdxdy = \frac{1}{2}(1 - cos1)$.)

Now, the clever student among you may be asking one or both of the following questions: Where have we used the Mean Value Theorem for Integrals in this part? And how do we know that integrating both sides of an inequality preserves the inequality? You may think that the answer to this second question is just obvious (think about areas under curves, or in this case, volumes under surfaces.) But was it actually stated anywhere in the book that integrating both sides of an inequality preserves the inequality? This is in fact where the MVT is being used. From the last inequality, subtract the left side from the right to get the following:

$$0 \leq \frac{sinx}{1+(xy)^4} - \frac{1}{2}sinx$$

Now you can apply the MVT:

$$0 \leq \iint_{[0,1]\times[0,1]} \left(\frac{sinx}{1+(xy)^4} - \frac{1}{2}sinx\right)dxdy$$

and finally arrive at the desired result:

$$0 \leq \iint_{[0,1]\times[0,1]} \frac{sinx}{1+(xy)^4}dxdy - \iint_{[0,1]\times[0,1]} \frac{1}{2}sinxdxdy$$

$$\iint_{[0,1]\times[0,1]} \frac{1}{2}sinxdxdy \leq \iint_{[0,1]\times[0,1]} \frac{sinx}{1+(xy)^4}dxdy$$