HOMEWORK 4

- Section 2.9 in the book: Exercises 10, 18, 22, 26, 28.

Problem 1. Consider the differential equation

$$\frac{dx}{dt} = |x|^{1/2}(1-x).$$

(a) Explain why for any $x_0 \in (-\infty, \infty)$ there exists a solution to the equation satisfying the initial condition $x(0) = x_0$, at least on some time interval containing 0.

(b) Give an example of a rectangle in the tx plane on which the existence and uniqueness (Picard) theorem does not apply. Justify your answer.

(c) Identify the equilibrium points.

(d) Draw a phase diagram and identify the stable and unstable points.

(e) Sketch the equilibrium solutions in the tx plane. These equilibrium solutions divide the tx plane into regions. Sketch at least one solution curve in each of these regions.

(f) For the particular solution with initial condition $x(0) = 0.43$, what is the limit $\lim_{t\to\infty} x(t)$?

Problem 2. Determine whether the Picard Theorem guarantees that the differential equation

$$\frac{dx}{dt} = \sqrt{x^2 - 9}$$

admits a unique solutions through the following points:

$a)$ $(-1, 1)$	$c)$ $(5, 3)$
$b)$ $(2, -3)$	$d)$ $(1, 4)$.