HOMEWORK 4

- Section 4.1 in the book: Exercises 22, 24, 30.
- Section 4.3 in the book: Exercises 24, 26, 32, 34, 36.

Problem 1. Show that for the differential equation

$$y'' + y = 0,$$

(a) there are infinitely many solutions obeying $y(0) = y(\pi) = 0$;
(b) there is exactly one solution obeying $y'(0) = 0$ and $y(\pi) = 1$;
(c) there are no solutions obeying $y(0) = 1$ and $y(\pi) = 1$.

Problem 2. Consider the equation

$$t^2 y'' + 7ty' + 5y = t \quad \text{for} \quad t > 0.$$

(a) Verify that $\phi_1(t) = t^{-1}$ is a solution to the associated homogeneous problem.
(b) Look for a solution to the inhomogeneous problem of the form $Y(t) = v(t)\phi_1(t)$. Plug this into the equation and prove that v' must satisfy a first order linear differential equation. Solve it and find Y.

Problem 3. Consider the equation

$$(1 - 2t - t^2)x'' + 2(1 + t)x' - 2x = 0.$$

(a) Verify that $\phi_1(t) = t + 1$ is a solution to the equation.
(b) Let ϕ_2 be a second solution to the differential equation so that $W(\phi_1, \phi_2)(0) = 1$. Use Abel's theorem to find the Wronskian determinant of ϕ_1 and ϕ_2 at all times $t \in (-1 - \sqrt{2}, -1 + \sqrt{2})$.
(c) Use part (b) to find all possible solutions ϕ_2 satisfying $W(\phi_1, \phi_2)(0) = 1$.

Problem 4. Consider the two functions

$$\phi_1(x) = x^2 \quad \text{and} \quad \phi_2(x) = x|x|.$$

(a) Show that ϕ_1 and ϕ_2 are linearly independent on $(-\infty, \infty)$.
(b) Show that $W(\phi_1, \phi_2)(x) = 0$ for every real number x.