First Name: ____________________ ID#____________________

Last Name: ____________________

Section: ____________________

$$= \begin{cases} 1a & \text{Tuesday with M. Bulkow} \\ 1b & \text{Thursday with M. Bulkow} \\ 1c & \text{Tuesday with L. Vera} \\ 1d & \text{Thursday with L. Vera} \\ 1e & \text{Tuesday with A. Mennen} \\ 1f & \text{Thursday with A. Mennen} \end{cases}$$

Rules.

- There are **FOUR** problems; ten points per problem.
- No calculators, computers, notes, books, crib-sheets,...
- Use the backs of the pages. There are two spare pages at the back.
- Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring,... Try to sit still.
- Turn off your cell-phone, pager,...

1	2	3	4	$\sum$

(1) (a) Finish this definition:

We say that $T : \mathbb{R}^m \to \mathbb{R}^n$ is a *linear transformation* if and only if ...

Now consider the linear system

$$\begin{bmatrix} 0 & 1 & 0 & 7 \\ 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 4 \\ 3 & 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \\ 12 \end{bmatrix}$$

(b) Find all solutions to this system by Gauss-Jordan elimination.

(c) Indicate the reduced row echelon form of the coefficient matrix.

(d) Is the coefficient matrix invertible? **YES/NO**?

(2) The following matrix is in reduced row echelon form:

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Fill in the missing numbers: A is a ______ $\times$ ______ matrix. It has rank ______.

(b) Indicate the pivots (by circling them).

(c) Is there a choice of $\vec{b}$ so that $A\vec{x} = \vec{b}$ has no solutions **YES/NO**?
If yes, give an example of such a $\vec{b}$.

(d) Is there a choice of $\vec{b}$ so that $A\vec{x} = \vec{b}$ has a unique solution **YES/NO**?
If yes, give an example of such a $\vec{b}$.

(e) Is there a choice of $\vec{b}$ so that $A\vec{x} = \vec{b}$ has infinitely many solutions **YES/NO**?
If yes, give an example of such a $\vec{b}$.

(3) Let R and T be linear transformations mapping $\mathbb{R}^2 \to \mathbb{R}^2$.

(a) R is reflection across the line through the points $(0,0)$ and $(1,1)$.
What is the matrix representing R?

(b) Given that

$$T(\left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]) = \left[\begin{smallmatrix} 3 \\ 5 \end{smallmatrix}\right] \quad \text{and} \quad T(\left[\begin{smallmatrix} 3 \\ 3 \end{smallmatrix}\right]) = \left[\begin{smallmatrix} 6 \\ 12 \end{smallmatrix}\right]$$

what is the matrix representing T?

(c) Find the matrix representing the composition $R \circ R$.

(d) Find the matrix representing the composition $R \circ R^{-1} \circ T \circ R$.

(e) Do R and T commute? **YES/NO**?

(4) The linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 4 & 6 & 5 \end{bmatrix}$$

(a) Determine A^{-1}.

(b) For each $p \in \mathbb{R}$ find $\vec{x} \in \mathbb{R}^3$ so that $T(\vec{x}) = \begin{bmatrix} p \\ 0 \\ p^2 \end{bmatrix}$.

Answer Key

1(b) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \\ 17 \\ 3 \end{bmatrix}$

1(c) the identity matrix

1(d) YES.

2(a) A is a 4×5 matrix. It has rank 3.

2(c) YES. $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

2(d) NO.

2(e) YES. $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

3(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

3(b) $\begin{bmatrix} 3 & -1 \\ 5 & -1 \end{bmatrix}$

3(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3(d) $\begin{bmatrix} -1 & 3 \\ -1 & 5 \end{bmatrix}$

3(e) NO.

4(a) $A^{-1} = \begin{bmatrix} 4 & 1 & -1 \\ -1 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix}$

4(b) $\begin{bmatrix} 4p - p^2 \\ -p \\ -2p + p^2 \end{bmatrix}$