HOMEWORK 4

Due on Wednesday, February 4th, in class.

Exercise 1. Solve exercises 1.5.2, 1.5.3, 1.5.6, 1.5.7, and 1.5.8.

Exercise 2. Let A be a non-empty set and consider the set of functions $f : A \to A$. Prove that the composition of such functions is commutative if and only if the cardinality of A is one.

Exercise 3. Define two internal laws of composition on $R = \mathbb{Z} \times \mathbb{Z}$ as follows

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$$
$$(a_1, a_2) \cdot (b_1, b_2) = (a_1 b_1 + 2a_2 b_2, a_1 b_2 + a_2 b_1).$$

1) Show that with these operations R is a commutative ring with unity and no zero divisors.
2) Define an order relation $\leq$ on R as follows: $(a_1, a_2) \leq (b_1, b_2)$ if $a_1 + a_2\sqrt{2} \leq b_1 + b_2\sqrt{2}$ in the usual sense on $\mathbb{R}$. Prove that this is an order relation on R and that with it, R is an ordered ring.