HOMEWORK 2

Due on Wednesday, January 21st, in class.

Exercise 1. How many equivalence relations can be defined on a set with three elements? How about on a set with four elements? Justify your answers.

Exercise 2. If the set A has $n \geq 1$ elements and the set B has $m \geq 1$ elements, show that there are m^n many functions from A to B.

Exercise 3. Let $f : A \to B$ and $g : B \to C$ be two functions. Prove that if $g \circ f$ is injective, then f is injective. Show (via an example) that $g \circ f$ can be injective without g being injective.

Exercise 4. Let $f : A \to B$ and $g : B \to C$ be two functions. Prove that if $g \circ f$ is surjective, then g is surjective. Show (via an example) that $g \circ f$ can be surjective without f being surjective.

Exercise 5. Do there exist functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x) - f(-x) = x^2 \quad \text{for all} \quad x \in \mathbb{R}?$$

Exercise 6. Determine the function $f : \mathbb{R} \to \mathbb{R}$ satisfying the following condition:

$$2f(x) + 3f(1-x) = 4x - 1 \quad \text{for all} \quad x \in \mathbb{R}.$$

Is f bijective? If yes, compute its inverse.

Exercise 7. Let

$$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = \begin{cases} 2x + 3 & \text{if} \quad x < 0 \\ x^2 + 3 & \text{if} \quad x \geq 0. \end{cases}$$

and

$$g : \mathbb{R} \to \mathbb{R}, \quad g(x) = x + 1.$$

Determine $f \circ g$ and $g \circ f$.

Exercise 8. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} 2x - 1 & \text{if} \quad x \leq 2 \\ x + 1 & \text{if} \quad x > 2. \end{cases}$$

Show that f is bijective and compute its inverse f^{-1}.

Exercise 9. Let $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = |x - 2| + 1$. Determine $f([1,3])$, $f([-1,3])$, $f([1,\infty))$, $f^{-1}(\{2\})$, $f^{-1}((-\infty,1])$, and $f^{-1}([1,3])$.

Exercise 10. Solve exercises 1.7.19, 1.7.20, 1.7.21, and 1.7.22 from the textbook. For exercise 1.7.22 find the inverse of f as well.