HOMEWORK 2

Exercise 1. (10 points) Show that $\sqrt{15}$ and $(2 + \sqrt{2})^{1/2}$ do not represent rational numbers.

Exercise 2. Let a, b, c be real numbers. Show that

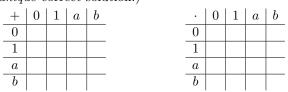
$$2ab \le a^2 + b^2$$

and

$$ab + bc + ca \le a^2 + b^2 + c^2.$$

Specify what axioms you are using at each step.

Exercise 3. Let $(F, +, \cdot)$ be a field with exactly four distinct elements $F = \{0, 1, a, b\}$ where 0 and 1 denote the identities for + and \cdot , respectively, and a, b denote the remaining two elements of F. Fill in the addition and multiplication tables below. Use the axioms to justify your solution. (Note that for each table entry there is a *unique* correct solution.)



Hint: Show that in the addition table each row and each column contains every element of F exactly once (as in Sudoku). Show that the same is true for the rows and columns of the multiplication table that are not identically zero.

Exercise 4. Solve exercises 3.5 and 3.6 from the textbook.

Exercise 5. Solve exercises 4.7, 4.8, 4.14, and 4.16 from the textbook.